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The vehicle routing problem and its intersection with cross-docking

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by

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ABSTRACT

To close a gap identified in the vehicle routing academic literature a theoretical link is established between the *Vehicle Routing Problem* and *Cross-Docking.* A model for the *vehicle routing problem with shipment consolidation* (VRPC), in which vehicles can consolidate cargo among one another at a customer's location, is presented. With shipment consolidation, vehicles can deliver product to a customer, transfer product to another vehicle, or both. Three main models are proposed: the *vehicle routing problem with shipment consolidation* (VRPC) which improves routing performance by allowing vehicles to consolidate cargo at any customer site; a metaheuristic to explore the effects of the VRPC over large-scale problems; and the *Vehicle Routing Problem with Shipment Consolidation and Time Windows* (VRPCTW) to further study the proposed concept under extended, more constrained circumstances. Computational experiments are developed and solved to optimality where possible using OPL and Java in conjunction with CPLEX and show that the proposed concept of shipment consolidation can provide significant savings in objective function value when compared to previously published models.

CHAPTER 1: INTRODUCTION

After its first study was published (Dantzig & Ramser, 1959), the Vehicle Routing Problem has continued to receive focused attention from scholars. Numerous variants and methodologies to solve this *NP-Hard* combinatorial optimization problem have been proposed. There still are many opportunities to further its current understanding. This problem has captivated the attention of various academic fields and that of practitioners in various industries that interact with logistics in one way or another. One largely important reason is the fact that logistics comprises an important part of the country's economy. The logistics component of supply chain management accounts for well over one trillion of the United States GDP. Sustaining a strong 60 percent of this amount, are trucking costs alone. Furthermore, in most of the last decade these transportation costs have exhibited an average growth of 5.5 percent, and its rates are already expected to be higher for 2017. In 2015 alone, motor carriers accounted for almost a staggering 600 billion dollars of total U.S. business logistics costs. All other modes of transportation (*i.e.* parcel, rail, air, water, and pipeline), if summed together, barely reach half that amount. Motors carriers even stood atop total inventory carrying costs for the U.S. business logistics industry, surpassing them by nearly 200 billion in 2015 (ATKearney & Penske, 2016).

However, transportation is not simply a monetary component of the economy. The human component is vastly important. The U.S. Bureau of Labor Statistics has disclosed that the estimate demand for truck drivers is closing in on 1.7 million persons (ATKearney & Penske, 2016). With average wage increases of merely 2 percent between 2010 and 2015, companies must find incentives to keep driver turnover under control. An example is how some publicly-traded firms show higher rates of increase in employee compensation than they do in total revenue. Previous studies have shown how better truck routing models can increase employee morale and decrease costs associated with driver turnover in significant proportions (Suzuki, Crum, & Pautsch, 2009).

Lastly, the impressive growth of B2C e-market has led various firms (*e.g.* Walmart, Costco) to change their strategies and make significant investments in information technology and mathematically-driven computer models to help leverage their operations in the benefit of all stakeholders. Examples of these issues are answers to questions such as: how do I fulfill this customer's order? Do we ship to store? Ship directly to the customer? Do we use on-hand inventory from a store nearby? Do we ship from a different store or warehouse further away? These are not queries that can be easily solved by a single employee within a reasonable amount of time. Furthermore, customers are asking for increased service levels exhibiting close-to-flawless quality. Thus, it appears safe to say that efficient computer models are in an ever-high demand.

Supply chain management and, more specifically, logistics must show that it can be a major cornerstone in the creation of value-adding activities, not simply a capacity to *move things* from a source to a customer. The industry and its representatives must converge in techniques adequate to support value-creating activities throughout their supply chains. Examples of current problems in transportation pertain to issues such as selecting a less efficient transportation mode, expediting ordinary shipments, miscalculating truck arrival/travel times, among others. One way to approach solutions to these inefficiencies is via better analytical models within the logistics knowledge domain.

In logistics, a widely studied model is that of the Vehicle Routing Problem, along with its special case, the *Capacitated Vehicle Routing Problem* (CVRP). Its objective is to minimize the total routing cost of identical vehicles of fixed capacity, starting and finishing at the depot, while each customer is visited exactly once to deliver or collect a fixed demand, without exceeding the capacity of vehicles. In a separate research stream, scholars have extensively studied *cross-docking* operations. Findings suggest cross-docking can significantly reduce costs and improve productivity (Soltani & Sadjadi, 2010). The objective of cross-docking is to consolidate shipments with a common delivery destination in a specialized warehouse with the intent to reduce transportation costs without sacrificing customer service.

Currently there is room to provide a new model that generalizes the CVRP with cross-docking. There are three characteristics which are noteworthy. First, the evident requirement of the CVRP stating that the items required by a customer must be placed on the same vehicle (no multiple deliveries are allowed). This particular shortcoming was addressed by what is now known as the CVRP with *split-delivery* (VRPSD). Second, the CVRP assumes that the fleet is comprised by unrealistically standardized vehicles. Finally, the cross-docking literature is yet to evaluate the effects of having vehicles transfer payload (commodities) throughout the network, relaxing the necessity of having formal cross-docking warehouses at various locations.

This essay aims to unfold the benefits of the proposed model of shipment consolidation in vehicle routing problems. With consolidation, vehicles can meet at any customer location and *i)* partially or completely fulfill the customer's demand, or *ii)* transfer commodities among one another. The approach has the potential to bring significant savings to the objective function's value (*e.g.* miles driven, fuel cost, time spent). This investigation also seeks to shed light on some of the condition under which the decision to consolidate shipments may be more worthwhile. Sequential interviews with supply chain practitioners, representatives of large TL and LTL transportation companies, were conducted to help determine some of the most delicate shortcomings of the VRPC. The outcomes of these meetings were used to refine the theoretical model and study whether the proposed concept of shipment consolidation is feasible under more realistic, constrained scenarios.

This document is organized as follows. First, a model for vehicle routing with shipment consolidation is proposed and studied by solving small-customer instances to optimality. Second, a metaheuristic is proposed to study the effects of the proposed concept on large problems. Finally, a more complex model is proposed and it includes variables regarding time scheduling, customer site consolidation feasibility, and added costs of consolidating.

CHAPTER 2: AN EXAMINATION OF THE INTERSECTION BETWEEN VEHICLE ROUTING AND CROSS-DOCKING

Introduction

Vehicle routing continues to receive attention from various stakeholders. Even though the *Great Recession* allegedly ended in 2009, the logistics industry continued to receive its aftermath for the following years. It was not until 2007 when the industry started to recover and shortly thereafter observed surprising revenue levels in 2014, when it reached $1.45 trillion. Logistics costs accounted for 8.5 percent of the United States GDP in 2012 totaling $1.33 trillion. Furthermore, transportation and Third-party logistics providers (3PLs) are expecting sustained growth throughout 2015 and 2016. Traditionally, the logistics and transportation industry is divided in logistic services, air and express delivery, freight rail, maritime, and trucking. The trucking industry alone accounted for a substantial $643 billion revenue stream in 2012, as per the United States American Trucking Association. Trending technological advancements are encouraging 3PLs to merge and increase their cloud-based service offerings to help leverage partners' performance. In addition, cross-docking and multimodal transportation are becoming increasingly attractive, and supply chain software is becoming ubiquitous. Railway operations are gaining all–time popularity rates and better ways to delivery products from a source to a customer in the supply chain are more important than ever. This does not constitute a simple trend and is unlikely to fade away. One key aspect to highlight is that, even though the logistics industry experienced significant growth, such a hike was due to a volume increase rather than a rate increase. This means that rates can become stagnant, which requires operations be carefully streamlined and vast efforts be placed in continuously improving logistics processes. Academics and practitioners need to aim efforts at the investigation of transportation efficiency and to push the threshold to provide better service rates without significant compromise in quality, cost, delivery capabilities, and flexibility, among other competitive priorities. One way to address these necessities is to account for product variety in vehicle routing and incorporating freight classes in analytical models. The much suggested increased complexity in mathematical models (Katok, 2011) will complicate their application but will simultaneously improve their value and favorable impact on daily operations. With numerous studies accounting the nuances that supply chain and logistics practitioners face in their business efforts, research can help increase business performance from various perspectives while contributing to the epistemology of our fields.

To that regard, a model is proposed model herein to improve the efficiency of logistics operations. It accounts for various elements used in an LTL shipment's freight class and uses these elements to optimize the way in which demand is delivered to customers. Specifically, factors of density (volume and weight) along with liability (value) of shipped products are accounted for and carefully implemented to maximize the usage of available fleet space and thereby minimize routing cost. The proposed model for the *vehicle routing problem with shipment consolidation* (VRPC) merges the literature in vehicle routing with that of cross-docking to propose a novel alternative that improves solution quality. Such model allows vehicles to transfer cargo among one another at any customer location (*i.e.* shipment consolidation). Whether the problem entails a heterogeneous or homogeneous fleet, various commodities, and several shipment factors or dimensions (hereinafter referred to as *facets*, *e.g.* volume, weight, value) the proposed model is suitable to readily identify solutions that can increase vehicle utilization and improve routing performance.

With the proposed model, supply chains can be designed to utilize shipment consolidation and take advantage of said improvements in vehicle deployment, while offering savings in aspects such as space utilization, labor–hours, vehicle maintenance, or fleet size.

TMS are becoming increasingly ubiquitous and are now at the forefront of supply chain planning and design. These systems can yield substantive savings in transportation costs. Furthermore, computer specifications have evolved to a point that allows a better integration of information systems and intelligent optimization aiding in a firm's decision-making capabilities, supported by rigorous mathematical models. All these systems aligned with efficient models that consider real-world constraints and variables to solve realistic problems under reasonable times are a necessity. The proposed model can aid to carefully evaluate decision scenarios that in the long term can improve logistics performance altogether.

Literature review

Vehicle routing problem

The Vehicle Routing Problem VRP has been studied extensively after its first publication by Dantzig & Ramser (1959), along with many different exact and approximate methodologies to solve it, such as branch-and-bound, cutting plane algorithms, heuristics, and metaheuristics (Michel Gendreau, Guertin, Potvin, & Taillard, 1999; Michel Gendreau, Hertz, & Laporte, 1994; Lysgaard, Letchford, & Eglese, 2004; Mitchell, 2002; Toth & Vigo, 2002a). It is a combinatorial optimization problem that exerts strong practical relevance and carries considerable difficulty. Essentially, the VRP is concerned with establishing an optimal route for an available vehicle fleet such that each vehicle departs from a depot, serves a set of customers, and returns to the depot. Each variant of the VRP will have various imposed constraints for said route construction. Some of the common constraints include limitations on delivery and collection (each vehicle can only perform either), vehicle capacity, time windows to visit each customer, a homogeneous (or heterogeneous) fleet of vehicles, stochastic or deterministic customer demands, split deliveries or single-visits to each customer, and symmetric or asymmetric distances (traversing an arc in one direction has a different cost than in the opposite direction).

The most studied version of the VRP is the Capacitated Vehicle Routing Problem (CVRP) (Toth & Vigo, 2014). The CVRP is comprised by a central depot, a set of *n* customers, and a cost matrix denoting the cost (generally length) of each pair of arcs. Each customer has a nonnegative demand, and a finite fleet of identical vehicles, based at the depot and with finite capacity *Q*, is available to serve the customer set. The objective is to find the optimal route such that: each route starts and ends at the depot; each customer is visited exactly once; the total demand of each route does not exceed *Q*; and the total routing cost is minimized (Cordeau, Gendreau, Laporte, Potvin, & Semet, 2002). Other variations to the CVRP include ***delivery and collection*** (*e.g.* Sankaran & Ubgade, 1994; Golden, Assad, & Wasil, 2001), in which some customers require the *delivery* of goods and, conversely, others require the *pickup* of goods. An example would be in the case of good distribution to retailers while performing reverse supply chain operations to recover defective items from the market. The routes generally only perform either of these two operations. A special case of the CVRP with delivery and collection is that in which *linehaul* (delivery) and *backhaul* (collection) customers can coexist in the same route. Under these settings, linehaul customers are served first to have an empty vehicle the moment in which the first backhaul is done. Similarly, the particular case called the CVRP with ***mixed deliveries and collections*** in which there are no difficulties associated with the loading and unloading of the vehicle and thus linehauls need not be served before backhauls (*e.g.* Min, 1989; Wade & Salhi, 2002). In this configuration, the vehicle capacity must be checked as each arc is traversed any capacity constraints are not violated. Another relevant case to the topic in question is the CVRP with ***Split Delivery*** or VRPSD (*e.g.* Archetti, Speranza, & Hertz, 2006; Dror, Laporte, & Trudeau, 1994; Dror & Trudeau, 1989; Frizzell & Giffin, 1995; Ho & Haugland, 2004). These problems are more specifically tailored to events in which the total demand of a customer exceeds the capacity of the vehicle, and thus more than a single visit becomes is imperative. In addition, the cost savings that can be obtained by partially delivering goods have proven to be considerable. In this setting, a customer's order can be fulfilled by different vehicles, each carrying a portion of the total order.

Cross-docking

Companies such as UPS, Toyota, and Walmart attribute great part of their success to efficient usage of cross-docking systems (Yu & Egbelu, 2008). Furthermore, with time-varying changes in oil prices, cross-docking is gaining outstanding popularity. Motor carriers, third-party logistics companies (3PLs) and shippers are incessantly facing difficult challenges to help leverage partner firms' performance. 3PLs are merging and increasing their cloud based service offerings, cross docking and multi modal transportation are becoming increasingly attractive, and supply chain software is turning ubiquitous.

Cross-docking is one way to decrease logistics costs and help leverage partners' performance. It is defined as "*receiving product from a supplier or manufacturer for several end destinations and consolidating this product with other suppliers' product for common final delivery destinations*" (Kinnear, 1997). It increases customer service performance and also outperforms traditional warehousing in inventory investment, storage space, handling cost and order cycle time, inventory movement, and cash flow (Kuo, 2013). Cross-docking consists of five basic *Distribution Center* (DC) operations: *receiving, sorting, storing, picking,* and *shipping*.

If these five elements are cooperating in tandem, cross-docking activities can reduce costs and improve productivity (Soltani & Sadjadi, 2010). However, unlike a DC, cross-docking facilities (*i.e. cross-docks*) typically store items for small periods of time. In an ideal scenario a cross-dock would have no inventory and picking operations would be negligible. However, this is realistically difficult to attain because nearly-impossible coordination would be required (*e.g.* all the orders of a single supplier might not arrive at the same time, thus temporary storage is required until all the necessary goods are available). Customers are requiring specialized products and services at an increasingly faster delivery rate. Shippers and Third-Party Logistics (3PL) companies are making an effort to increase customer service while keeping costs stagnant. All five basic functions of a DC will generally have room for improvement, especially in cases with greater product flow. However, the less handling and storage that a DC has to perform, the better the operational performance of such facility (Yu & Egbelu, 2008). Therefore, we can conclude that storage and picking are generally the two most expensive tasks in both cross-docking and DC operations.

A traditional cross-dock typically operates as depicted in Figure 1. First, *inbound trucks* arrive at a *receiving dock* where they can unload the items. These items can be of various sizes, shapes, and packing. Ideally, trucks would experience no queue and they could be served immediately. At this point, items are scanned and inspected, often weighed and labeled. Second, they are placed on conveyer belts and sortation systems and subsequently taken to appropriate sections based on their destinations. The most beneficial scenario is that in which there is no need to temporarily store any items (Items can be immediately shipped out). However, if storage is required then these items are set on hold in corresponding areas. Subsequently, when the time is right items are picked from their storage area and prepared for shipment out of the cross-dock.

Finally, the items are transferred to the appropriate location on the *shipping docks*, where they are placed in *outbound trucks* and leave toward their next destination.

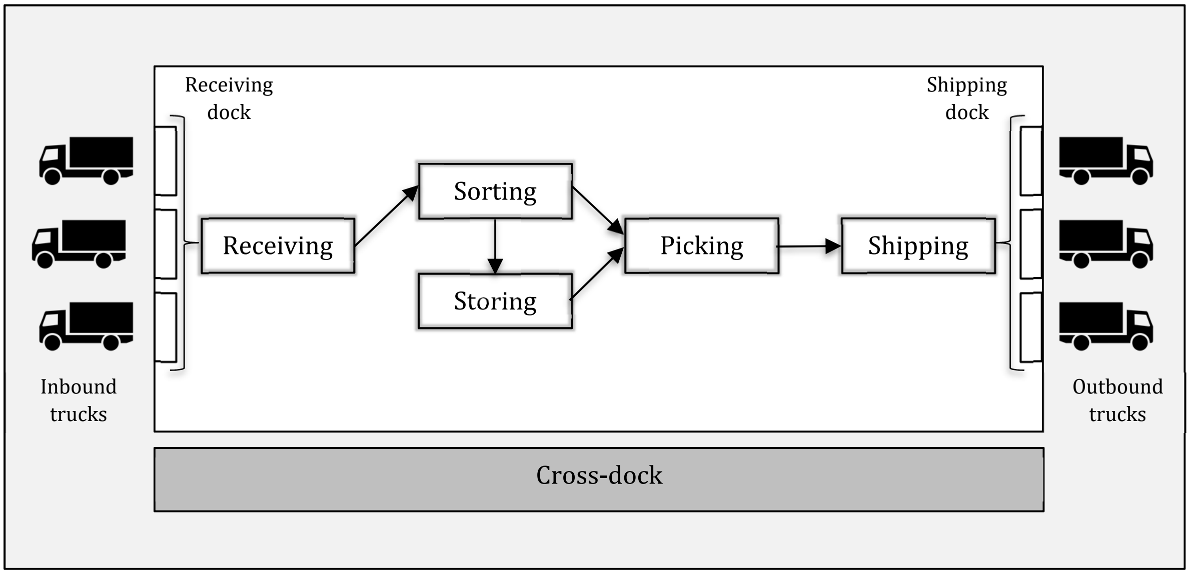

Figure 1 - General flow of a cross-dock

The objective of cross-docking is to serve as an order-consolidation warehouse where the time items spend held in inventory is minimized. Naturally, the larger the firm the more likely it is to have an increased inventory turnover and, by transitivity, lower cross–docking costs. The cross-docking model can serve to consolidate product before *last mile* delivery and can be used to switch transportation mode. It is expected that once a large-enough material handling volume is attained, cross–docking becomes increasingly cumbersome and many challenges arise. There are different technologies that can be set forth to improve its performance such as hardware (*e.g.* RFID, GPS, barcode readers) and software (*e.g.* WMS, ERPs, custom software modules) but the most important is the symmetry of information and secure yet fast and easy access to the necessary information such that the parties involves can adequately plan the use of resources. Cross–docks need be properly designed to adequately handle the product flow intended for it and to do this, information transparency is of especial importance. The focus of conventional cross-docking models is the minimization of two main aspects of time: that which products spend at the cross-dock and truck idle-time (unloading and loading). Typically, cross-docking optimization models will assume that sorting and storage need not be optimized. Warehousing is allowed but physical destinations inside the DC are commonly not established. Furthermore, trucks generally stop once at either the receiving or the shipping dock when *all* of the relevant cargo is either unloaded or loaded, and no revisits are allowed. Long-term (indefinite) storage is not available. Temporary storage is possible but not ideal (the time-minimization objective function would usually avoid these circumstances).

Even though considerable research has studied cross-docking and also vehicle routing with loading constraints (Galbreth, Hill, & Handley, 2008; Michel Gendreau, Iori, Laporte, & Martello, 2006; Kum Khiong Yang, Balakrishnan, & Chun Hung Cheng, 2010; Kuo, 2013; Soltani & Sadjadi, 2010; Tarantilis, Zachariadis, & Kiranoudis, 2009; Vahdani & Zandieh, 2010; Wei, Zhang, & Lim, 2014; Yu & Egbelu, 2008; Zhu, Qin, Lim, & Wang, 2012), very few actually attempted to intersect both areas in a dynamic fashion. Wen et al. (2009) presented a model labeled the Vehicle Routing Problem with Cross-Docking in which a set of homogeneous vehicles is used to fulfill the orders of a customer set via a cross-dock. Their model uses a cross-dock to consolidate orders while minimizing storage time and considering customers' time windows. Lee et al. (2006) published a similar model, in which multiple suppliers and customers must be visited within an allotted time window. However, these publications are limited to a single cross-dock at fixed coordinates.

The model presented in this study is the *vehicle routing problem with shipment consolidation* (VRPC). This problem consists of single set of customers that require the delivery of a finite set of different commodities, each of varying dimensions and a finite fleet of vehicles with fixed capacity, whose tours start and end at a central depot. The VRPC has the particularity that vehicles can consolidate cargo mid-route. To this end, vehicles can meet at any customer location and *i)* partially or completely fulfill the customer's demand, or *ii)* transfer cargo among one another. This process will be hereinafter referred to as *shipment consolidation*. While this approach relaxes the constraint that each customer can only be visited by one vehicle it has the potential to considerably decrease the objective function's value or total cost (*e.g.* miles traveled).

Intersection of the VRP with Cross-docking

Scant studies have delved into the possibility of intersecting the CVRP with cross-docking in a dynamic fashion. Most studies thus far have merged vehicle routing to and from the cross-dock. However, they are both treated in the traditional fashion: vehicles move cargo and the cross-dock transfers such cargo from one medium to another while the cross-dock is at a fixed location. However, the proposed model attempts to evaluate a different approach and thereby challenge the paradigm that a cross-dock need be a brick-and-mortar facility with fixed coordinates. By relaxing these assumptions and allowing the model to propose dynamic cross-docking points significant savings can be attained in the form of total distance covered, which can further be generalized to reduced fuel consumption, fleet size, vehicle maintenance, labor hours, among others.

To introduce the proposed model, a revision of the conventional *Capacitated Vehicle Routing Problem* (CVRP) is appropriate. The CVRP consists of the schedule of sequential visits to distribute goods from a single *depot*, denoted as zero (0), to a set of n nodes, typically referred to as *customers*, such that $N = \{1, 2, \dots, n\}$. Each customer $i \in N$ must has a specific *demand*, given by a scalar $q_i \geq 0$ while noting that the depot's demand $q_0 := 0$. The goods must be delivered to the customers using a *fleet* of $|K|$ *homogeneous* vehicles, such that $K = \{1, 2, \dots, |K|\}$, each of which has a standard capacity $Q > 0$. The CVRP consists of routing each vehicle to serve a subset of customers $S \subseteq N$ to depart from the *depot,* visit each customer in S once, and finally return to the depot. The CVRP can be set in either a directed or an undirected graph depending on whether traversing an arc in a certain direction has a different cost than traversing it in the opposite direction, or if such cost is equal regardless of the direction in which it is traveled. We will focus in the more general case of the directed graph. The *vertices* or nodes are the set $V = \{0\} \cup N = \{0, 1, 2, \dots, n\}$ and the arc set $A = \{(i, j) \in V \times V: i \neq j\}$ having c_{ij} be the cost of traveling from node i to node j for $(i, j) \in A$. The CVRP is thus uniquely defined by the complete digraph $G = (V, A, c_{ij}, q_i)$ along with the vehicle fleet K of size $|K|$ and the vehicle capacity Q. A *route* or *tour* is a sequence $r = (i_0, i_1, i_2, \dots, i_s, i_{s+1})$ where $i_0 = i_{S+1} = 0$ in which the subset $S = \{i_1, \dots, i_S\} \subseteq N$ of customers is visited. Each route r has a cost $c(r) = \sum_{i=0}^{S+1} c_{i,i+1}$ associated. A route in which *a)* the capacity constraint $q(s) := \sum_{i \in S} q_i \leq Q$ is not violated; and *b)* each customer is visited only once (*i.e.* $i_j \neq i_k \forall\ 1 \leq j \leq k \leq s$); is said to be a *feasible cluster* $S \subseteq N$. A *solution* to the CVRP consists of $|K|$ feasible routes, dictating the tours of $|K|$ available vehicles. A *feasible solution* will be the routes $r_1, r_2, \dots, r_{|k|}$ along with their corresponding clusters $S_1, S_2, \dots, S_{|k|}$ in which all the routes are feasible and the clusters form a partition of N.

The CVRP becomes the solution of two simultaneous and interconnected tasks: i) partitioning the set of customers N into feasible clusters $S_1, S_2, \dots, S_{|k|}$; and ii) routing each vehicle $k \in K$ along $\{0\} \cup S_k$. The complexity of the graph G is defined by $|A| = n(n + 1)$ and therefore we can conclude that it is of the form $O(n^2)$. The following is the basic formulation of the CVRP. First, define $S \subseteq V$ as an arbitrary subset of vertices. Second, let $\delta^-(S) = \{(i, j) \in$

$A: i \notin S, j \in S\}$ be the in-arcs of S and let $\delta^+(S) = \{(i,j) \in A: i \in S, j \notin S\}$ be the out-arcs of S. Third, let $A(S) = \{(i,j) \in A: i,j \in S\}$ the subset of arcs connecting all vertices in S. Furthermore, given a customer subset $S \subseteq N$, let $\gamma(s)$ be the minimum number of vehicle routes needed to serve S. The lower bound of $\gamma(s)$ is given by the expression $\lceil q(s)/Q \rceil$. It is now possible to proceed to establish a general form of the CVRP in a Mixed Integer Programming (MIP) model with polynomial variables with respect to n = |N| and |K|.

(1.1) $$\text{minimize} \sum_{(i,j)\in A} c_{ij} \cdot x_{ij}$$

subject to

(1.2) $$\sum_{i\in\delta^+(i)} x_{ij} = 1 \qquad \forall\, i \in N$$

(1.3) $$\sum_{i\in\delta^-(j)} x_{ij} = 1 \qquad \forall\, i \in N$$

(1.4) $$\sum_{j\in\delta^+(0)} x_{0j} = |K| \qquad \forall\, j \in N$$

(1.5) $$\sum_{(i,j)\in\delta^+(S)} x_{ij} \geq \gamma(S) \qquad \forall\, S \subseteq N, S \neq \varnothing$$

(1.6) $$x_{ij} \in \{0,1\} \qquad \forall\,(i,j) \in A$$

The objective of the model (1.1) is the overall minimization of routing costs. The set of constraints (1.2) and (1.3) guarantee that each customer vertex in a route is only connected to two other vertices; in other words, that each customer is visited only once. Additionally, the set of constraints (1.4) ensure that exactly $|K|$ routes are constructed. These constraints can readily be replaced with inequalities of the form "≤" if the fleet size is larger than needed or $|K| > \gamma(N)$. Next, constraints (1.5) are both *capacity cut constraints* and *Subtour Elimination Constraints* (SEC) which eliminate tours in which the vehicle capacity is not violated and that any isolated tours not connected to the depot are also deprecated. Another approach is the MTZ formulation (Miller, Tucker, & Zemlin, 1960), which replaces equations (1.4) and (1.5) and introduces the additional variables $u = (u_1, \ldots, u_n)^T$. These u_i variables represent the cumulative demand delivered by the vehicle after it arrives to customer $i \in N$. The MTZ formulation introduces the following two sets of constraints as the SEC constraints and the capacity cut constraints, respectively as follows:

$$u_i - u_j + Q \cdot x_{ij} \leq Q - q_j \qquad \forall (i,j) \in A(N) \tag{1.7}$$

$$q_i \leq u_i \leq Q \qquad \forall i \in N \tag{1.8}$$

The advantage of the MTZ formulation is that it involves $n^2 + n$ constraints and $n^2 + 2n$ variables. However, even though this formulation would have $O(n^2)$ complexity, its linear relaxation for the MIP model produces a significantly weaker lower bound (Toth & Vigo, 2014).

A model for vehicle routing with shipment consolidation

In this essay, mid-route commodity consolidation is presented as a form of cross-docking. The objective corresponds with the goals of lean supply chain management: smaller volumes of more visible inventories that are delivered faster and more frequently (Van Belle, Valckenaers, & Cattrysse, 2012). The goal is therefore to capitalize on the capabilities of more frequent and faster deliveries of cross-docking. The concept consists of turning every customer site into a bay where a simple consolidation of shipments can be performed. The intended goal is to reduce total distance covered by vehicles with *less-than-truckload* cargos. One of the main benefits of the proposed model of VRPC is the lack of need for a specific location where cross-docking operations are conducted. Consider the example in which two vehicles, k_1 and k_2, with *less-than-truckload* payloads are traveling in parallel to visit two customers that are not far from each other. Furthermore, assume that the total cargo of both vehicles can be consolidated in a single truck (*i.e.* $u_1 + u_2 \leq Q$). In this case, there should exist a point, not far from k_1 and k_2's locations, where either u_1 is loaded onto u_2, or conversely should that be more favorable. This way, only one vehicle would continue traveling and the other could return to the depot.

Model definition

The problem is defined by G = (V, E), a complete graph where $A = N \cup \{0\} = \{0,1,2, \dots, n\}$ is the set of nodes, and $E = \{(i,j) | (i,j) \in A \times A\}$ is the set of edges joining each pair of nodes. Let $N = \{1,2, \dots, |N|\}$ be the set of customers and $|N| = n$, and $\{0\}$ be the central depot. The distance of each arc $(i,j) \in E$ is denoted by d_{ij}. Let $p \in P = \{1, 2, \dots, |P|\}$ represent the various commodities (products) customers require, and $q_i^p = \{q_0^p, q_1^p, \dots, q_n^p\}$ for $i \in A, p \in P$ be the set of non-negative customer demands, with $q_0^p = 0 \; \forall \, p \in P$. The fleet is comprised of available vehicles $k \in K = \{1,2, \dots, |K|\}$, where each vehicle k has a capacity of Q_k^f for each facet $f \in F = \{1, \dots, |\mathrm{F}|\}$. Facets are the dimensions modeled in the problem and correspond to any characteristic or dimension that a carrier deems important like volume, weight, value, risk, among others. Each commodity p has a corresponding multiplier δ_p^f, which is a measure of the magnitude of each $p \in P$ for a facet $f \; E \; F$ (e.g. Let product p_1 be pillows, facet f_0 be weight, and facet f_1 be volume. Then δ_1^0 could be 2 *lb* and $\delta_1^1 = 1.7 \; ft^3$). The objective is, like in the traditional capacitated vehicle routing problem, the minimization of the total distance traveled by the fleet of vehicles, such that: (a) each vehicle route must start and end at the depot; (b) each customer's demand must be completely fulfilled; (c) the payload of any vehicle traveling through any arc (i,j) cannot exceed Q_k^f for some facet f.

The proposed model for the *vehicle routing problem with shipment consolidation (VRPC)* can now be established as follows:

(2.1) $$\text{minimize} \sum_{(i,j)\in A,k\in K} d_{ij}^{k} \cdot x_{ij}^{k} \qquad \text{(VRPC)}$$

subject to

(2.2) $$\sum_{i\in A,k\in K} x_{ij}^{k} \geq 1 \qquad \forall\ j \in N$$

(2.3) $$\sum_{j\in A} x_{0j}^{k} \leq 1 \qquad \forall\ k \in K$$

(2.4) $$\sum_{i\in A} x_{im}^{k} - \sum_{j\in A} x_{mj}^{k} = 0 \qquad \forall\ m \in A, k \in K$$

(2.5) $$\sum_{(i,j)\in S} x_{ij}^{k} \leq |S| - 1 \qquad \forall\ S \subseteq N, S \neq \emptyset, k \in K$$

(2.6) $$\sum_{i\in N,p\in P,k\in K} u_{i0}^{kp} = 0$$

(2.7) $$\sum_{i\in A,k\in K} u_{im}^{kp} - \sum_{j\in A,k\in K} u_{mj}^{kp} = q_{m}^{p} \qquad \forall\ m \in N, p \in P$$

(2.8) $$u_{ij}^{kp} \leq M \cdot x_{ij}^{k} \qquad \forall\ (i,j) \in A, k \in K, p \in P$$

(2.9) $$\sum_{p\in P} u_{ij}^{kp} \cdot \delta_{p}^{f} \leq Q_{k}^{f} \qquad \forall\ (i,j) \in A, k \in K, f \in F$$

(2.10) $$x_{ij}^{k} \in \{0,1\}$$

(2.11) $$u_{ij}^{kp} \in \mathrm{N}^{0} = \{0,1,2,\ldots\}$$

Objective function (2.1) minimizes total distance traveled through the selected arcs with the chosen vehicles. The set of constraints (2.2) ensure that each customer is visited at least once. (2.3) ensure that each vehicle departs the depot at most once. This way, the model can also identify whether there are any vehicles to spare. (2.4) are the classical network flow constraints;

this set of constraints ensure that the number of vehicles arriving to each customer equals the number of vehicles departing from it. (2.5) correspond to the subtour elimination constraints. The set of constraints (2.6) guarantees that there is no inflow of product to the depot. Constraints (2.7) enable shipment consolidation in the system. These constraints seek the maintenance of product flow and demand fulfillment throughout the network. Essentially, the product *p* that arrives to customer *m*'s location on vehicle *k* has to: *1)* entirely (or partially) fulfill the m^{th} customer's demand of said product (q_m^p), or depart in the same (or another) vehicle. The set (2.8) link the two decision variables *u* and *x* with a large integer *M* adjusted based on the problem's characteristics. Constraints (2.9) pertain to the capacity cut constraints, ensuring that the load of any vehicle *k* traversing the arc (*i,j*) does not exceed *k'*s capacity in terms of a certain facet *f*. Finally, the set *x* in (2.10) is a binary decision variable, which takes on a value of *one* if vehicle *k* travels arc (*i,j*), *zero* otherwise; and *u* in (2.11) is a nonnegative, integer-valued decision variable that represents the unit load of product *p* in vehicle *k* over the arc (*i,j*).

The real value of VRPC

The most important aspect of the VRPC is its ability to account for the various characteristics of the commodities vehicles carry. Carriers prefer to minimize the number of less-than-truckload shipments. One way to do this is to consolidate product heading to nearby destinations, such that the distance a vehicle travels up to capacity is maximized. However, not always is the product homogeneous as carriers expand their market niche. Furthermore, there are numerous restrictions that prevent these ideal scenarios from taking place in real life. Shipping managers struggle with product consolidation as the types of products being carried becomes more diverse. Motor carriers, third-party logistics companies (3PLs) and shippers are incessantly facing difficult challenges to help leverage partner firms' performance, such as shortness of slack resources to undertake complicated research projects. 3PLs are merging and increasing their cloud-based service offerings, cross-docking and multimodal transportation are becoming increasingly attractive, and supply chain software is becoming more ubiquitous. One way to address these upcoming situations is to account for commodity variety in vehicle routing and incorporating freight classes in analytical models. A set of facets *F* can include elements from various aspects that are used to determine an LTL shipment's freight class. The factors that typically determine a freight class are **density** (weight, length, height): dimensions of length, width, height, and volume are used to determine density and help classify an item in a specific class; **stowability:** some items cannot be loaded together and certain hazardous materials must be transported according to specific regulations; **handling**: whether freight required especial mechanical equipment or special attention in loading and carrying; and **liability**: which translates to probability of freight theft or damage or damage to other cargo, likelihood of combustion or explosion, perishable cargo, among others. The proposed model of VRPC can account for these classes by incorporating set *F,* which includes the necessary factors for each problem. For example, say shipper *s* must restrict the economic value of a vehicle's cargo to be no greater than *a* dollars at any given point due to insurance requirements. Furthermore, let *i* (a customer of *s*) require the delivery of a piece of equipment *e* that, however small, has a value $v \approx a$ (The vehicle cannot carry more items because of value restrictions, even though it has plenty of unused cargo space). Normally, *s* would have to make a single-customer tour to deliver *e* to *i.* After the delivery, *s* would be left with an empty truck somewhere in its supply network ready to return to the depot. With VRPC however, *s* can have that vehicle resupplied at *i*'s location with some other mix of cargo and continue the delivery process, thus avoiding a trip to the depot. The VRPC can create a feasible product mix in each vehicle that accounts for all the different restrictions that shippers face in their daily conduction of business; for example, to not carry more than *a* dollars at any given point. Also, consider the case in which p_1 = *bricks* and p_2 = *pillows*. A vehicle's weight capacity would be reached with fewer bricks than it would with pillows. Similarly, a vehicle could be filled with a smaller number of pillows than it would of bricks. The VRPC intends to exploit all these differences in the best interest of performance.

Transformation from VRPC to VRPSD

The proposed model of VRPC can readily be adapted to prevent shipment consolidation from taking place and behave akin to a traditional capacitated *vehicle routing problem with split deliveries* (VRPSD) instead, in which no shipment consolidation takes place but a customer may be visited by multiple vehicles (*Split delivery*). This functionality is achieved by adding set of constraints (3). The added constraints ensure that the flow of each vehicle traversing the network is always decreasing. Normally, if two vehicles consolidate shipments along the network, the cargo of one vehicle will increase just as that of the other will decrease, after a visit to a customer location. Therefore, constraints (3) prevent any shipment consolidation from taking place and force the proposed model to behave like a traditional VRPSD.

$$\sum_{i \in A} u_{im}^{kp} - \sum_{j \in A} u_{mj}^{kp} \geq 0 \qquad \forall\, m \in N, p \in P, k \in K \tag{3}$$

Proposition 1: *the optimal solution value obtained with the VRPC model will be at least as good as that obtained with a VRPSD model.*

The VRPC not only accounts for heterogeneous product characteristics but also increases the flexibility of routing processes by enabling an enhanced usage of the available fleet. The backbone of the proposed model is the possibility to perform product transfer across any set of vehicles $K' \subseteq K$ at a customer location. Thus, any number of vehicles can visit each customer and either pickup or transfer product. However, if no vehicle is transferring cargo then a multiple-vehicle visit to any customer n_i turns into a split delivery. The full potential of the VRPC will be discussed in subsequent sections. However, under certain conditions, in which the typology of the network does not realize any savings by consolidating shipments mid-route, the optimal solution will have the form of a VRPSD in which no consolidation is possible. This is particularly true when the demand is homogeneous (*i.e.* the product to be delivered is standard) and there is no account for differences in terms of maximum weight, maximum volume, among other dimensions, that a vehicle can carry at any given point. Therefore, the VRPC can solve a single-product simplified problem instance of the VRP and obtain an optimal solution which is at least as good as that obtained with the VRPSD. Some models of VRPSD (Claudia Archetti & Speranza, 2012; Dror & Trudeau, 1989; Mullaseril, Dror, & Leung, 1997; Sierksma & Tijssen, 1998) include (4), or a variant thereof, to guarantee product flow and demand fulfillment throughout the network. In (4) y_{ik} is a nonnegative real-valued decision variable denoting the percentage of the demand of customer i that is fulfilled by vehicle k. Equation (4) thus ensures that each customer's demand is met by all vehicles visiting i.

$$\sum_{k} y_{ik} = q_i \qquad \forall\, i \in N \tag{4}$$

Now, to see how the VRPC includes (4), let (2.7) of the VRPC be rewritten in such a way as to represent a single-product problem ($|P| = 1$), which will resemble a traditional VRPSD. Since there is only one type of product to be carried, the capacity of the vehicle must be monitored in *only* one facet (e.g. volume, weight, units, etc.) and there is no need for p, as follows:

$$\sum_{i \in A, k \in K} u'^{k}_{im} - \sum_{j \in A, k \in K} u'^{k}_{mj} = q_m \qquad \forall\, m \in N \tag{5}$$

Where the generalized version of u_{ij}^{kp}, u'^{k}_{ij} is a nonnegative decision variable that represents the general load of vehicle k over the arc (i,j). Now, start from (3) and replace u_{ij}^{kp} with its single-product counterpart, u'^{k}_{ij} to obtain set of constraints (6). Set of equations (7) is the translation of u'^{k}_{ij} from (6) to y_{ik}, which is relatively straight-forward as both variables represent a vehicle's nonnegative payload. Finally, (8) represents the resulting extension of (7) while summing across all vehicles in the fleet, which after replacing the right-hand side with (5), results in (9).

$$\sum_{i \in A} u'^{k}_{im} - \sum_{j \in A} u'^{k}_{mj} \geq 0 \qquad \forall\, m \in N \tag{6}$$

$$y_{ik} = \sum_{m \in N, m \neq i} u'^{k}_{mi} - \sum_{m \in N, m \neq i} u'^{k}_{im} \geq 0 \qquad \text{for some } k \in N \tag{7}$$

$$\sum_{k \in K} y_{ik} = \sum_{k \in K, m \in N} u'^{k}_{mi} - \sum_{k \in K, m \in N} u'^{k}_{im} \tag{8}$$

$$\sum_{k} y_{mk} = q_m \qquad \forall\, m \in N \tag{9}$$

Thus, it is possible to identify how (9) is embedded within the VRPC. Since (9) is a set of constraints equal to (4), which characterize the VRPSD then we can conclude that (3) will ensure that each vehicle fulfills some $q_m \geq 0 \; for \; m \in N$ without transferring part of its payload onto a different vehicle, thereby ensuring no mid-route shipment consolidation.

Preliminary experiments

To the best of the author's knowledge, a model like the VRPC has not emerged in the literature up to when this essay was produced. Some articles have studied similar models, such as the vehicle routing problem with split deliveries (VRPSD). However, the VRPC is differentiated in that it allows shipment consolidation from one vehicle to another as they meet at a customer's site. Furthermore, studies have seldom elaborated on the typology of small instances (*e.g.* those in which $n \leq 10$) that can be easily solved to optimality (See Archetti et al., 2006). To test the feasibility of the VRPC and tentative benefits of shipment consolidation, four initial test instances were produced. The sample problem instances were named with the following pattern: the first two characters indicate problem size (5n signifies 5 customers or $n=5$). Next, the number of facets is presented (3f for three facets). Subsequently, a capital letter K indicates heterogeneous fleet, whilst a lower-case k implies a homogeneous fleet of vehicles. Finally, the third instance presented has the added word "Cluster" which implies that some customer locations were artificially clustered.

$$\theta_i^f = \frac{\sum_{p \in P} q_i^p \delta_f^p}{Q_{\hat{k}}^f} \tag{10}$$

$$\Theta^f = \frac{\sum_{i \in N} \sum_{p \in P} \delta_f^p \cdot q_i^p}{\sum_{k \in K} Q_k^f} \tag{11}$$

Table 1 describes the generated problem instances. The customer capacity utilization ratio θ_i^f in (10) represents the ratio of the total demand of customer i divided by the capacity available in a standard vehicle k, all in terms of facet f. When $\theta_i^f > 1$, more than one vehicle is required to deliver the demand of customer i. If $1 < \theta_i^f < 2$ then one large vehicle, if available, could carry the total weight required by customer i (Because a large vehicle is assumed to have twice the capacity of a standard one). Similarly, if $\theta_i^f > 2$ then not even a large vehicle could singlehandedly supply the entire demand of customer i. In some cases, the demand of a certain customer i may be such that it does not exceed the capacity of a vehicle in terms of one facet, but it may do so in terms of another. Some problem instances set forth herein use $P = \{0, 1, 2\}$ or a subset thereof. Similarly, Θ^f in (11) is the total demand to total available capacity (System-wide) ratio in terms of facet *f*. From here follows that if some $\Theta^f > 1$ then the instance is infeasible because there is not enough total capacity across all vehicles to transport the required product by all customers. The instances included in Table 1 describe some characteristics of the instances included in this analysis. These instances were solved to optimality in an Intel Core i7 CPU with 8 GB of memory and running Windows 10. The VRPC was coded in OPL and ran on IBM Ilog CPLEX with a branch and bound algorithm. Unless otherwise noted in table1, $\theta_i^f < 1 \; \forall \, i$.

Table 1 - Preliminary test instances for the VRPC

No.	Instance	θ_i^f	Fleet (*K*)	Facets (*F*)	Products (*P*)	Θ^f
1	5n3f-K	$\theta_1^2 \approx 1.62$ $\theta_3^2 \approx 2.01$ $\theta_4^0 \approx 1.71$ $\theta_5^1 \approx 2.74$	1 large (2X capacity of standard vehicle), 3 standard	$F = \{0, 1, 2\}$	$P = \{0, 1, 2\}$	$\Theta^0 \approx .80$ $\Theta^1 \approx .77$ $\Theta^2 \approx .80$
2	6n2f-k	$\theta_5^0 \approx 2.10$ $\theta_5^1 \approx 2.09$ $\theta_6^0 \approx 2.18$ $\theta_6^1 \approx 2.01$	5 standard	$F = \{0, 1\}$	$P = \{0, 1\}$	$\Theta^0 \approx 1.00$ $\Theta^1 \approx .99$
3	6n2f-k-Cluster	$\theta_4^1 \approx 1.02$ $\theta_5^0 \approx 1.29$ $\theta_5^1 \approx 1.99$ $\theta_6^0 \approx 2.15$ $\theta_6^1 \approx 1.01$	5 standard	$F = \{0, 1\}$	$P = \{0, 1\}$	$\Theta^0 \approx 1.00$ $\Theta^1 \approx 1.00$
4	7n2f-k	$\theta_4^0 \approx 1.01$ $\theta_6^0 \approx 1.76$ $\theta_6^1 \approx 2.01$	5 standard	$F = \{0, 1\}$	$P = \{0, 1\}$	$\Theta^0 \approx .99$ $\Theta^1 \approx .98$

In addition to the capacity utilization ratios mentioned in Table 1, the following parameters were used to define the problems. The three facets included in the problems were $f = \{0, 1, 2\}$. The fleet is determined by the set $K = \{0, 1\}$, in which all vehicles are of equal capacity (homogeneous) or there is one larger vehicle (heterogeneous). Product magnitudes δ_p^f were set such that each commodity *p* was significantly larger than the other two, in terms of one facet *f*. Each customer was given a fixed set of coordinates and the cost matrix c_{ij} was obtained based on Euclidean distances amongst each customer pair. In addition, the large vehicle was given a cost factor of 1.5 that was used to account for any additional expenditures that a large vehicle would incur when compared to the smaller vehicles. Thus $c_{ij}^k = CostFactor \cdot c_{ij}$ depending on vehicle's *k* cost factor.

Results of preliminary experiments

Table 2 includes the results of each optimal solution when solved using VRPSD (split delivery without shipment consolidation) and in the third column, the objective function value when the same instance is solved using the proposed model of VRPC (with shipment consolidation). Finally, the last column shows the comparisons of the optimal solution value of both models. As expected, the proposed model of VRPC performed better than the split-delivery only model. A graphical comparison of both models follows.

Table 2 - Optimal solution values for the VRPC vs. VRPSD

Instance	VRPSD	VRPC	Gap
5n3f-K	322.81	298.95	-7.39%
6n2f-k	232.54	229.17	-1.45%
6n2f-k-Cluster	319.11	306.60	-3.92%
7n2f-k	314.99	302.07	-4.10%

Figure 2 illustrates the optimal solution to 5n3f-K obtained through VRPSD. As evidenced by the graph, all the vertices are visited by at least two vehicles. In addition, vehicle k_2 is performing the longest route by visiting every node except 1. With 7.39% savings, Figure 3 shows the same instance solved with the proposed model VRPC. In this case, vehicle k_2 is traveling a shorter distance (by not visiting customer 2), which is only possible by the shipment consolidation that is taking place at customer 3 (k_2 transfers 328 item of p_1 to k_3 so k_3 alone can fulfill the demand of customer 2). Moreover, in Figure 2 vehicle k_1 must visit {5,4} alongside k_2. However, with VRPC this is no longer necessary as k_1 makes a single-customer tour to 5 carrying p_1 = 814 units to fulfill the demand of 5 (200) and to restock k_2 from 1386 to 2000. This enables k_2 to visit 4 by itself, thus reducing total distance traveled.

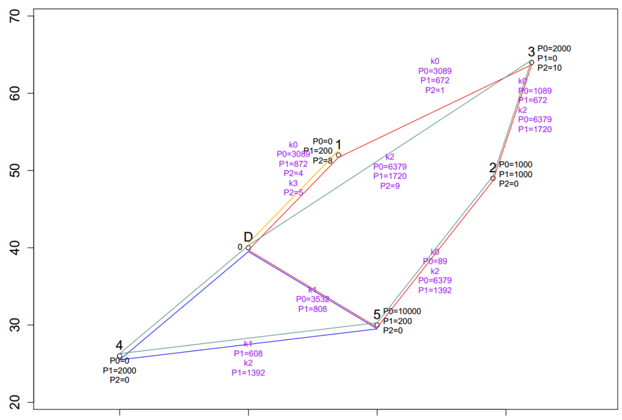

Figure 2 5n3f-K with VRPSD

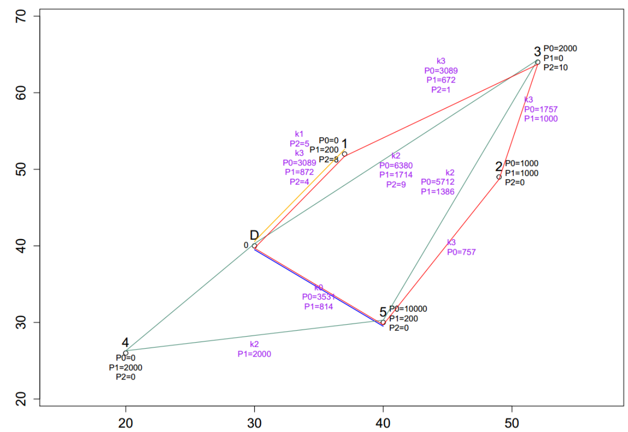

Figure 3 5n3f-K with VRPC

Figure 4 depicts 6n2f-k's optimal solution with VRPSD. In this case, both solutions are surprisingly similar. The difference lies in Figure 5 in which consolidation takes place by restocking p_1 from 862 to 912 in k_3 at customer 6's location. This subtle change only results in 1.45% savings in distance traveled. However, this exemplifies how VRPC can produce savings and increased efficiency with minor alterations to traditional routes.

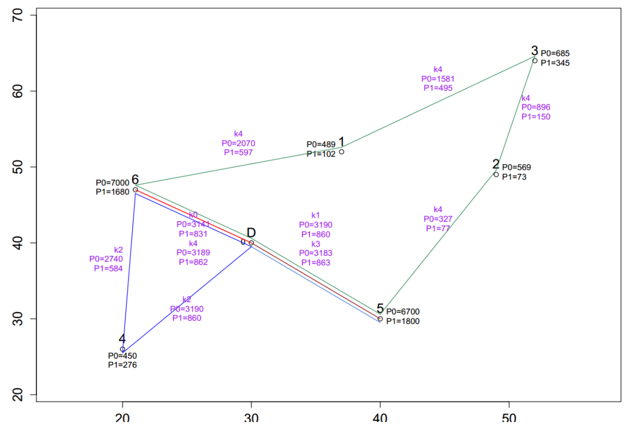

Figure 4 6n2f-k with VRPSD

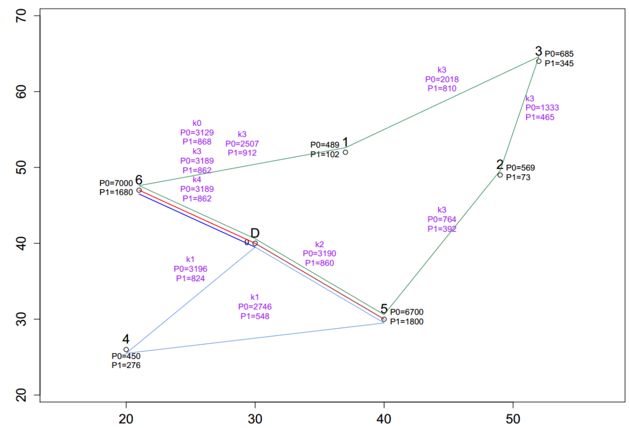

Figure 5 6n2f-k with VRPC

Instance 6n2f-k-Cluster is illustrated in Figure 6 with VRPSD and in Figure 7 with VRPC. This heterogeneous-fleet, 2-facet, 6-customer problem is differentiated because nodes 2 and 6 are clustered at the right side of the coordinate plane, with customer 1 located somewhere between the customer cluster and the depot. Figure 6 shows how without dynamic cross-docking, vehicle k_1 must travel across the four-vertex polygon comprised of {0, 5, 6, 1}. With VRPC, Figure 7 changes the routing such that *i)* k_1 is restocked at customer 6's location as its load of p_1 increases from 862 to 1086; and *ii)* k_2 increases its cargo of p_0 at node 4 such that it can visit customer 1 and satisfy its product demand with no longer the need for a split delivery.

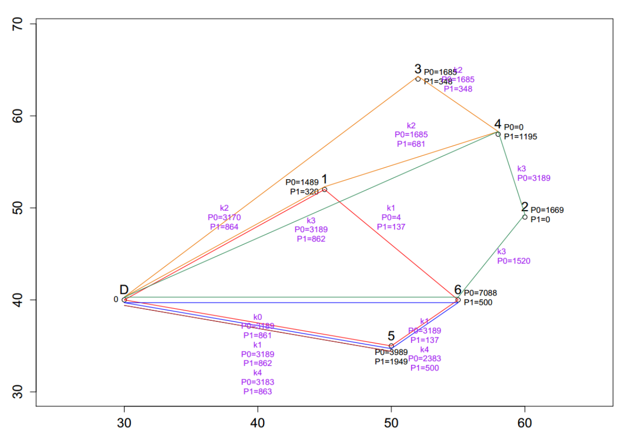

Figure 6 6n2f-k-Cluster with VRPSD

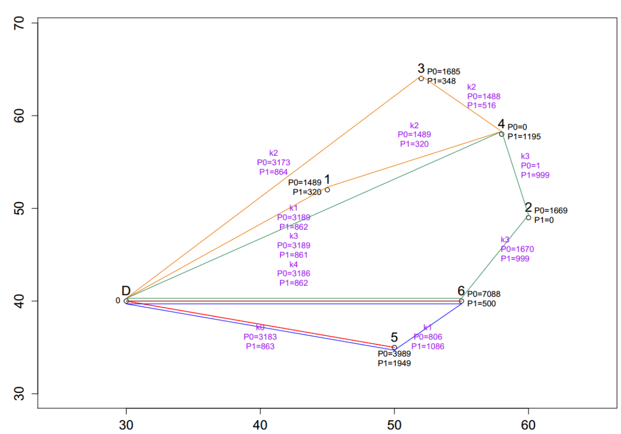

Figure 7 6n2f-k-Cluster with VRPC

Finally, Figure 8 and Figure 9 show the sample instance 7n2f-k, which includes seven customers and a homogeneous fleet of vehicles. The two-product and two-facet scenario is maintained. After running the proposed model of VRPC (Figure 9) a consolidation point is created at customer 4. Here, the cargo of vehicle k_4 is increased from p_1=877 to 1008. This operation allows k_4 to do the tour {0, 4, 5, 2, 3} in contrast to {0, 3, 2, 4} with the VRPSD (Figure 8). By adding node 5 to k_4's route k_1 no longer has to visit 5 and thus considerable distance is saved (4.10%).

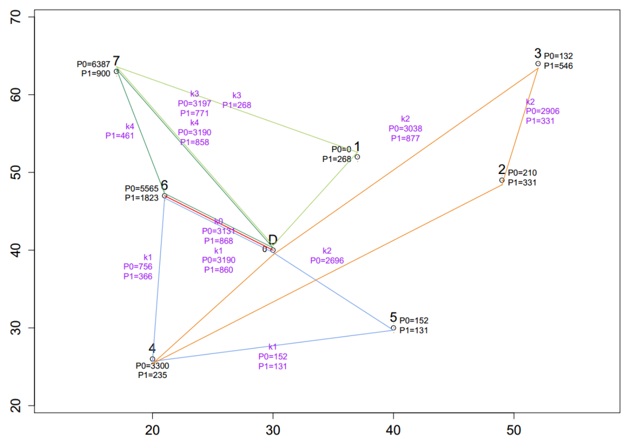

Figure 8 7n2f-k VRPSD

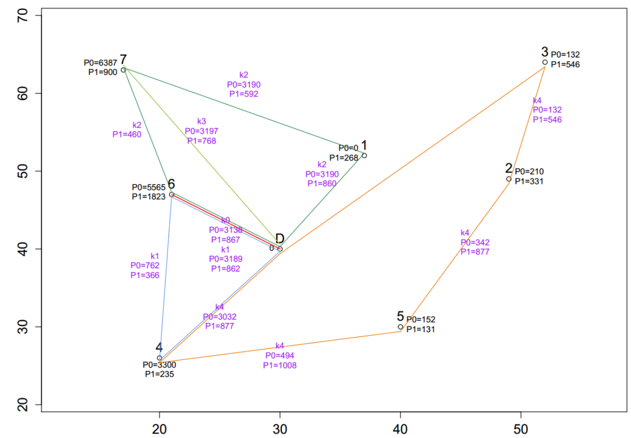

Figure 9 7n2f-k VRPC

The experiments herein mentioned briefly provide evidence as to the performance of the proposed model for the VRPC. By adding possible shipment consolidation points, the convex hull of feasible solutions includes more alternatives to solve the routing problem, while using similar resources and offering the possibility for considerable savings. In some cases, such as 6n2f-k, there are only slight changes made to the routes. However, the power of VRPC resides in the fact that even if a certain problem cannot benefit from consolidation, the optimal solution will be at least as good as any obtained with a previously demonstrated model to have excellent performance, such as VRPSD.

Computational experiments

To analyze the circumstances under which the proposed model for the VRPC performs better, the experimental design of Table 3 is proposed. The VRPC is a new variant of the vehicle routing problem and it represents a multi-commodity, multi-dimension, heterogeneous fleet, split-delivery routing problem, with mid-route shipment consolidation. Furthermore, the VRPC includes *facets,* or various measures of each commodity from multiple dimensions (*e.g.* weight, volume, value). Thus, there are no, to the best of the author's knowledge, available instances to use as a benchmark. For this reason, the instances used in this study were randomly generated, as follows. First, the typology of the network may affect the viability of transferring commodities across vehicles. Shipment consolidation may be particularly beneficial in cases where customers are somewhat clustered, or in those in which few customers are scattered away, while most of the customers are located somewhat close to one another. The variance of the arc length was manipulated across high and low levels. Second, businesses with fleets of divergent capacities can take advantage of larger vehicles, by using them as a mobile replenishment depot that can *reload* smaller, cheaper to run, vehicles as they traverse the network. Two levels have been defined based on this assumption: a homogeneous fleet wherein all vehicles have the same capacity; and a heterogeneous kind in which the fleet has exactly one vehicle with twice the capacity of the others. Third, studies have shown how patterns in customer demand can dramatically affect the performance of some models. Under this assumption, each customer's demand variance of each product is altered across two levels. Finally, to assess the impact of the size of the customer base, the number of nodes in the problem is changed. However, due to computational limitations, only small cardinalities of the customer set (|N| = 5 through 10) were solvable to optimality within reasonable amounts of processing time. Commodity magnitudes δ_p^f were set such that each commodity $p \in P = \{0,1\}$ was larger than the other, in terms of one facet $f \in F = \{0,1\}$. Thus, $2\delta_0^0 = \delta_0^1$ and $\delta_1^0 = 2\delta_1^1$. Finally, vehicle capacities $Q_k^f = \{Q_k^0, Q_k^1\}$ were set to {600, 600} for standard vehicles, while for large vehicles *k'* the capacity was $Q_{k'}^f = 2Q_k^f$.

The full-factorial experimental design has a total of 96 combinations (6 x 2 x 2 x 2 x 2) of the variables of interest. For each combination, 15 random problems were generated following normally distributed random variables with the corresponding parameters from Table 3. This translates to a total of 1’440 different problems. These problems were then solved with both the proposed VRPC model and the VRPSD (split deliveries are allowed but commodities cannot be consolidated across vehicles). This design raised the total number of problems to 2’880, out of which 2’874 were solved (6 problems in one combination of the variables of interest triggered a failsafe in the program code to prevent excessive solution times). Each problem was generated in Java (JDK and JRE 1.8) and solved using IBM Ilog CPLEX version 12.6.3. The experiments were conducted in a quad-core Intel Core i7 machine with 16 GB of memory running Windows 10. Some of the Java and CPLEX parameters were adjusted to maximize computational resource utilization and keep the solution time within reasonable limits.

Table 3 - Experimental design for the VRPC

Variable		Levels	Values
Problem size		6 levels	$\|N\| = 5$ $\|N\| = 6$ $\|N\| = 7$ $\|N\| = 8$ $\|N\| = 9$ $\|N\| = 10$
Arc length variance		Low High	$\mu = 1{,}000; \sigma = 50$ $\mu = 1{,}000; \sigma = 1'000$
Fleet type		Homogeneous Heterogeneous	Standard vehicles only Standard vehicles plus one large vehicle
Demand variance	Product 1	Low High	$\mu = 100; \sigma = 20$ $\mu = 100; \sigma = 200$
	Product 2	Low High	$\mu = 100; \sigma = 20$ $\mu = 100; \sigma = 200$

Results

The objective of computational testing is to evaluate the benefits obtained via the proposed concept of shipment consolidation (VRPC) against a traditional model of vehicle routing with split deliveries (VRPSD). Table 4 describes the different combinations of variables of interest (cells), aggregated by problem size. That is, each cell represents the results obtained over *all* the six levels of the *problem size* variable.

Table 4 - Performance of the proposed VRPC against the VRPSD, aggregated by problem size

Experimental variables					**VRPSD**		**VRPC**			
Cell	**σ(arc length)**	**σ(demand P_1)**	**σ(demand P_2)**	**Fleet**	**Time (Min)**	**Average SD**	**Time (Min)**	**Average SD**	**Average XD**	**Average Gap**
1	50	200	200	0	3.89	5.19	0.91	5.20	2.31	**-0.28%**
2	50	200	200	1	1.81	4.58	2.08	4.07	2.12	**-1.70%**
3	50	200	20	0	1.91	4.48	0.79	4.78	1.52	**-0.04%**
4	50	200	20	1	1.40	3.92	0.95	3.58	1.26	**-0.53%**
5	50	20	200	0	2.32	4.56	1.41	4.59	1.23	**-0.47%**
6	50	20	200	1	0.43	3.88	0.35	3.71	1.41	**-0.54%**
7	50	20	20	0	2.82	3.87	2.13	3.92	0.62	0.22%
8	50	20	20	1	0.18	3.20	0.06	3.10	0.46	**-0.44%**
9	1000	200	200	0	9.09	5.01	5.75	4.94	1.53	**-1.35%**
10*	1000	200	200	1	6.77	4.29	0.88	4.10	1.53	**-5.94%**
11	1000	200	20	0	6.57	4.18	4.56	4.49	0.80	**-1.11%**
12*	1000	200	20	1	2.99	3.60	0.68	3.49	0.79	**-3.50%**
13	1000	20	200	0	4.70	4.18	2.27	4.37	1.03	**-0.97%**
14*	1000	20	200	1	2.69	3.30	0.57	3.36	1.03	**-3.52%**
15	1000	20	20	0	2.36	2.96	1.05	3.46	0.36	**-0.58%**
16	1000	20	20	1	0.19	2.30	0.04	2.18	0.21	**-1.08%**
Total					**3.13**	**3.97**	**1.53**	**3.96**	**1.14**	**-1.36%**

Notes: SD and XD is the number of nodes in which a split delivery and product consolidation takes place, respectively.
A more negative gap implies better performance of the VRPC *vs.* the VRPSD
Fleet is equal to one when the generated problem had a heterogeneous fleet, homogeneous otherwise
* top three-performing combinations (cells) of the full-factorial experimental design

Savings potential

Results on Table 4 highlight the savings potential that shipment consolidation can entail. The *Gap* column is the average percent change observed when using the VRPC *vs.* using the VRPSD (A more negative gap value indicates larger savings due to the VRPC). Aggregating the entire results, across all 2'880 problems solved, showed an average improvement of the objective value of 1.5 percent due to the VRPC. This finding suggests that, on average, it is more beneficial to implement shipment consolidation and improve the fleet's utilization. The best-performing cells in the experimental design were: 10, 14, and 2. These combinations revealed an average improvement in the objective function value (lower value) of 6, 3.5, and 3.5 percent, respectively over 90 different problems solved in each of them. These cells appear to have higher variances in common. They all have highly varying arc lengths and, the demands of either product 1 or 2, or both. Furthermore, all these instances belong to cells with heterogeneous fleets. This suggests that the more scattered the customers and the more variant their demands are, coupled with a heterogeneous fleet, the higher the savings potential of the VRPC and its mid-route shipment consolidation capability.

Split delivery and consolidation behavior

The average number of consolidation sites (Avg XD or, the number of customer sites where payload consolidation occurs) is slightly greater than one (1.14) across all problems. This signals that transferring payload from one vehicle to another, in at least one opportunity, during all the routes in the fleet, can offer important savings. While the VRPC does rely on transferring payload across vehicles at a customer location, using such model does not appear to change the number of times a customer is visited on average. This is supported by the fact that the average number of split deliveries (Average SD) does not vary significantly across the two models

(Paired *t*-test $t = -0.88376$, df = 959, *p*-value = 0.1885). While some of those deliveries may involve shipment consolidation, the average number of split deliveries remains statistically comparable. See Figure 10 below.

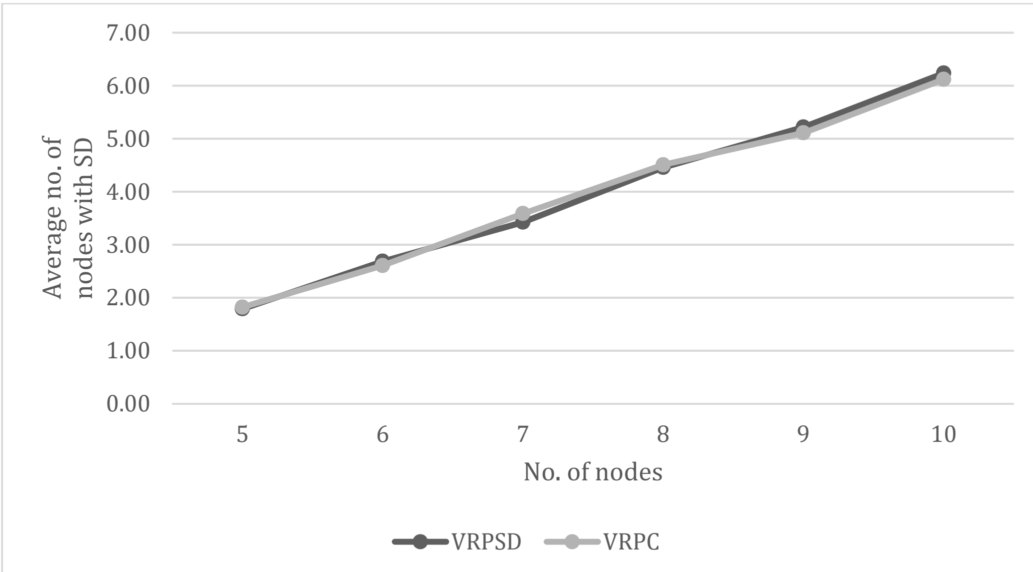

Figure 10 - Number of split deliveries (SD) in the VRPC *vs.* the VRPSD

Consolidation, however, seemed to be positively correlated to the problem size. See Figure 11, next. As shown by the results grouped by problem size, the number of customers wherein payload transfer across vehicles takes places seems to increase as a factor of the number of nodes in the network. This outlines the possibly greater benefits of implementing the proposed VRPC model on larger networks of customers.

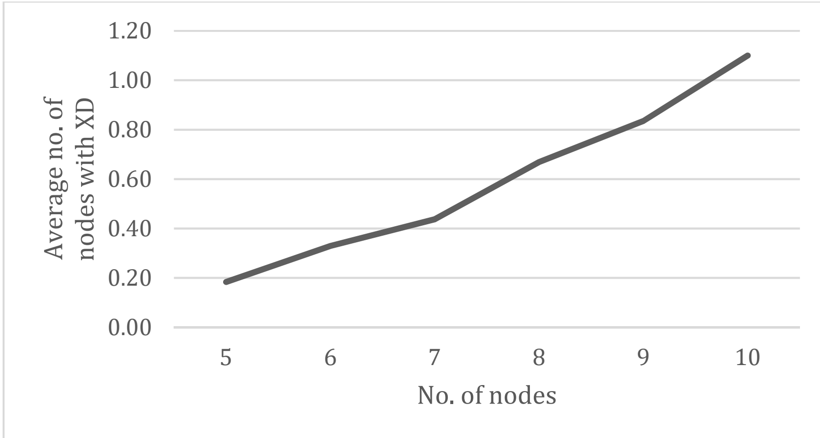

Figure 11 - Number of nodes with mid-route shipment consolidation (XD)

Computational time

Table 5 supports the expected time complexity of the problems. The computational times required to reach optimal solutions increases alongside the number of nodes in the network. This prevented further analyses using larger networks in which potentially better savings could be obtained. A time limit of 30 minutes was set on algorithm runtime, at which point its execution was terminated. If no optimal solution was reached, then a new problem was generated, and the algorithms were restarted.

Table 5 - Average computation time per model

Nodes	VRPSD	VRPC
5	0.00	0.00
6	0.02	0.01
7	0.22	0.04
8	1.24	0.36
9	6.93	2.59
10	10.41	6.18
Total	**3.13**	**1.53**

Discussion

Results from the numerical experiments support expected benefits accrued due to the VRPC. Reduced problem sizes attained nearly four percent savings in objective function value and overall savings of 0.7 percent across all cells (See Table 14 in the Appendix for full results grouped by problem size). More importantly, is that accrued savings from the VRPC appear to increase continuously with problem size. Figure 12 depicts a possibly greater objective function benefit as the number of nodes in the network increases. This evidence suggests that even larger logistics networks can gain even more benefits more implementing a mid-route shipment consolidation approach.

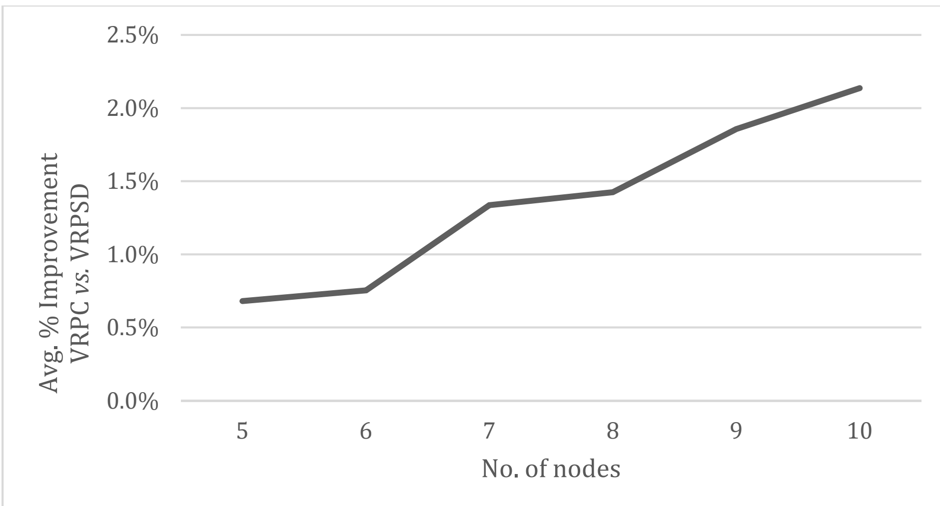

Figure 12 - Average improvement of the VRPC *vs.* VRPSD by problem size

Another effect worth noting is that not all cells in the experimental design perform equally. Figure 13 shows the generally best-performing instances across all cells. Interestingly, benefits of mid-route shipment consolidation continues to increase as does the size of the customer base. The clear pitfall of this process is the fact that these problems become computationally greedy as of very small network sizes (no. of nodes) thus complicating the discovery of optimal solutions. The largest benefits observed occurred with cell 10 (high variance of the: arc length, product 1's demand, product 2's demand; and a heterogeneous fleet). This finding suggests that the less clustered the network and the more diverse the demand pattern and the individual vehicle capacity, the more savings that can be expected via the implementation of the proposed model for the VRPC.

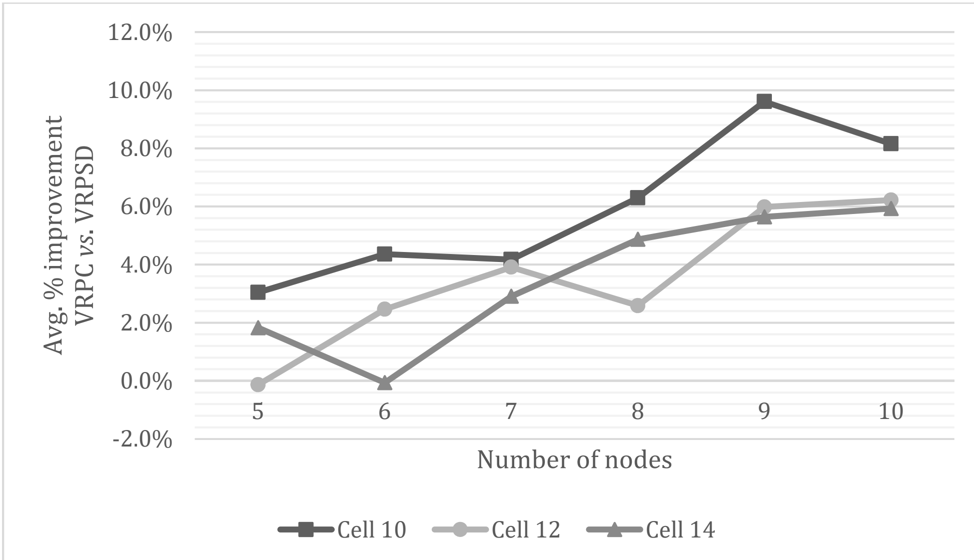

Figure 13 - Average savings through the VRPC *vs.* VRPSD in the top three cells of the experimental design

In the present design, the heterogeneous fleet was comprised, in the interest of simplicity, of one large vehicle with twice the capacity of the others. However, potential savings could be extended further by fleets of largely heterogeneous capacities. Further studies can analyze various combinations of fleet capacity distribution and derive better recommendations for a fleet mix that can gain even better performance from a concept such as mid-route shipment consolidation.

Conclusion

This study presented a new model for the Vehicle Routing Problem with Shipment Consolidation (VRPC), which implements the concept of mid-route shipment consolidation as a way for vehicles to transfer payload among one another at any customer location. Over 2000 problems for the VRPC were randomly generated and solved to optimality using a commercial solver. These random problems were solved with the VRPC and a traditional model that does not allow the transferring of payload across vehicles. Results suggest the VRPC is a promising concept that has the potential to improve the objective function value not only with small networks but even more so as the number of customers in the network increases. An important limitation was linked to the computational time available to run the necessary experiments. Further studies can adopt heuristics or other methodologies to gauge the performance of the proposed concept of mid-route shipment consolidation on larger, more realistic problems. The contribution presented herein is concerned with the introduction of this beneficial logistics concept that has the potential to save resources by re-thinking the way in which vehicles are routed.

Shipment consolidation is a concept better suited for firms or 3PLs with distribution networks that involve multiple stops along its routes (Mostly operating in the LTL segment of transportation). The fact that consolidation requires material handling at customer sites, firms with a private fleet and its own delivery sites can take especial advantage of this concept. In these cases, contractual barriers are likely to be easier to resolve and liability issues would be more efficiently addressed.

Final remarks

The proposed model (CVRP) merges the literature in vehicle routing problem with cross-docking in the form of mid-route shipment consolidation. The VRPC has shown evidence to perform at least as well as other previously known models such as VRPSD. In addition, its application can realize savings by performing subtle changes in fleet routing as simple as transferring payload across vehicles throughout the customer network. Conducted experiments show promising results. However, this study is not free of limitations.

First, the variables analyzed in this essay are limited. Further studies could study the effect of: *i)* Types and number of commodities; *ii)* Number of facets; *iii)* Customer demand or location patterns that can lead to more promising results.

Another interesting aspect of VRPC is the possible capacity utilization improvements for problems in which at least one of $\theta_i^f > 2$. For example, companies that transport high-value assets may want their fleet to carry no more than a certain amount of cargo, based on its economic value (when some $f_i = value$), even if such cargo only utilizes a small fraction of the available space. Under these circumstances, a vehicle could potentially delivery high-value items and, after its initial deployment, be restocked and utilized to serve other distant points in the network. These cases could take further advantage of varying fleet sizes wherein large vehicles may travel shorter distances but restock smaller vehicles along their way.

Another important application of the VRPC is for both urban logistics and disaster recovery logistics. Many authors have investigated different options to serve customers with small demand requirements in densely populated urban areas (Crainic, Ricciardi, & Storchi, 2004; Quak & de Koster, 2009). Similarly, other authors have studied different ways to deliver water to emergency shelter in a post-disaster or disaster-recovery situation (Nolz, Doerner, & Hartl, 2010; Ozdamar & Demir, 2012; Suzuki, 2012b). These cases often employ heterogeneous fleet of transportation modes such as dump trucks, pick-ups, bicycles, and even donkeys. These models may cut-off important sections of the solution domain and could take full advantage of VRPC by opening room for previously unexplored solutions.

Challenges associated with shipment consolidation at customers' locations

Performing shipment consolidation at a customer's location is not without challenges. There are several problematic issues associated with this approach. However, they only convey a path to elaborate further extensions of this work. Especial equipment (*e.g.* forklifts, cranes) or qualified personnel, among others, are resources that may not be readily available at customers' locations. For this reason, extra training and resource investments might be the initial barrier to the proposed models. However, in the long run, the VRPC should be more beneficial. One way to overcome this challenge is to develop portable equipment that can be carried inside the vehicles. Another solution could be to establish business partnerships with other firms that can rent their equipment or facilities on individual basis.

Liability issues arise the moment the cargo is manipulated. Cargo is generally sealed to prevent any unnecessary handling and to minimize risk. If any group of interested parties accepts shipment consolidation along the vehicles' routes, then proper stakeholders need to take adequate measures to prevent abrupt increases in risk. For this, all parties must be properly informed and risk-mitigating safeguards must be carefully put in place.

Customer sites with limited spatial availability may deem the proposed concept as infeasible, particularly if their turnover ratios are high and their facilities are highly congested. Nonetheless, given a shortage in available space *in-situ*, vehicles can find a nearby area with sufficient space to consolidate packages with negligible impact to overall costs. Furthermore, the quantity of items being consolidated could be constrained.

Man-hours are unlikely to be negligible. Consolidating shipments at a customer site will have an effect on wages since operators will have to spend time transferring shipment packages from one vehicle to another. This will impact operating costs but the result is expected to outweigh any additional costs in the long run. These are simply initial challenges in the horizon of the proposed model VRPC. However, the benefits of exploring this understudied logistics alternative may propose substantial benefits in the long run and can provide further insight in the way logistics operations are currently conducted.

Time-window consistency is an important aspect of vehicle routing. The scholarly community has also studied the VRP with ***time windows***, a variant of the CVRP that includes a given time interval during which customers must be served. Multiple studies have been published to date (Fisher, Joernsten, & Madsen, 1997; Kanoh & Tsukahara, 2008; Kolen, Rinnooy Kan, & Trienekens, 1987; Li, Tian, & Leung, 2010; Nagata & Braysy, 2009; Ohlmann & Thomas, 2007; Suzuki, 2012a). Timing and scheduling is a key aspect of this problem. One limitation of the VRPC presented herein is the lack of schedule coordination. It is worthwhile to investigate whether vehicles that consolidate shipments must wait disproportionate amounts of time at the customer's site. Possible solutions to this problem can be to drop off the transferable cargo at the customer's location to keep until the next vehicle collects it. However, this would be contingent upon the customer's ability and will to do so, which will likely come at a cost.

CHAPTER 3: A METAHEURISTIC FOR THE CAPACITATED VEHICLE ROUTING PROBLEM WITH MID-ROUTE SHIPMENT CONSOLIDATION

Introduction

The proposed model of *vehicle routing problem with shipment consolidation* (VRPC) merges the literature in vehicle routing with that of cross–docking to propose a novel alternative that improves solution quality. The model’s ability to allow vehicles to transfer cargo among one another at any customer location can be an important feature enabling previously infeasible solutions to the same problem while saving costs. The VRPC is highly relevant to the logistics sector as it can consider a heterogeneous fleet, multiple commodities, and several shipment factors (hereinafter referred to as *facets*, *e.g.* density, value, stowability, handling). The main purpose of this model is to increase vehicle utilization, resource efficiency, and ultimately improve routing performance. The VRPC has shown positive results under circumstances limited to small-problem instances given the lack of computational resources that can solve a problem to optimality, under a reasonable amount of time. Supply chain and logistics practitioners can improve their operations with a metaheuristic that can take full advantage of the VRPC while solving more realistic solutions under an acceptable time window.

No metaheuristic to solve the VRPC has been proposed in the past. This is an unexplored subdomain of the vehicle routing literature. By proposing an algorithm that intelligently discards solutions and proposes good-quality ones, scholars can continue to shed light on the best alternatives to solve these difficult combinatorial optimization problems. The proposed algorithm (VRPCSA) involves the exploration and generation of various solution neighborhoods applicable for the VRPC. The multi-commodity, multi-facet nature of the VRPC can also serve as a steppingstone to tackle similar problems from a metaheuristic perspective (*e.g.* hybrid problems that involve facility-location or risk-evaluation problem along with vehicle routing). Which responds to a call for further investigation in this field (Griffis, Bell, & Closs, 2012). Though the Vehicle Routing Problem with Split Deliveries (VRPSD) has been studied under a multi-commodity lens (Claudia Archetti et al., 2014), these commodities have been assumed to have equal physical magnitudes. Furthermore, vehicle capacities are represented by a single integer value. In addition, other studies have analyzed which types of problems are better suited to be solved with a VRPSD model, such as those whose customers have certain demand characteristics Archetti et al. (2008). However, the VRPC has shown promising results that outweigh previous models such as the VRPSD. Thereby calls for further investigation through more robust analyses, especially for problem instances with larger, more realistic, network cardinalities are in warrant. The algorithm presented herein targets a more extensive study of the VRPC in an effort to corroborate previous findings with small problem instances, to obtain more generalizable results, and to present a metaheuristic that can be used to solve both the VRPC and other similar problems such as the VRPSD and multi-commodity CVRP.

Literature Review

The intersection of vehicle routing and cross–docking has been established as an important literary gap in logistics research. With more astute models aiding analytical decision-making in the corporate marketplace, businesses can help leverage their operations and fulfill customer requirements while increasing resource utilization efficacy. The proposed model for the *vehicle routing problem with shipment consolidation* (VRPC) builds a relevant application of freight classes and product diversity to previously known models. This innovative model enables shipment consolidation at any customer site. It supports a heterogeneous or homogeneous fleet of vehicles, various commodities, any shipment facets (*e.g.* volume, weight, value, handling) and can improve previous solutions with minor investments in additional resources. In the previous chapter, a mathematical model was proposed to solve the vehicle routing problem with dynamic cross–docking. However, limitedly large instances can be solved to optimality given the *NP–hard* nature of the *Traveling Salesman Problem* embedded in the VRP. Several metaheuristic algorithms have shown promising advancements in the solution of the Capacitated Vehicle Routing Problem (CVRP) within an acceptable time frame while offering negligible optimality gaps. Even though metaheuristics cannot guarantee optimality (unless infinite resources of computational power or time are available) these algorithms can solve more realistic problems within time windows that resemble those which businesses would be willing to have. More importantly, metaheuristics have the capability to solve problem instances with hundreds of customers in only a few seconds, while the best branch–and–cut–and–price algorithms (currently regarded as the state–of–the–art solvers for these types of problems) can take up to several hours (Fukasawa et al., 2006; Lysgaard et al., 2004; Pecin, Pessoa, Poggi, & Uchoa, 2014).

Given the promising nature of the proposed model of VRPC, a metaheuristic becomes imperative to corroborate its performance under larger problems with a significantly large solution domain. The VRPC has its roots in cross–docking literature and that of the Capacitated Vehicle Routing Problem with Split Deliveries (VRPSD). Thus, these are two natural subfields to include in this investigation. Previous models have developed various metaheuristics for the VRPSD. Archetti et al. (2006) presented SPLITABU, an algorithm for the homogeneous–fleet CVRP consisting of three stages: the generation of an initial feasible solution based on the

GENIUS algorithm (M Gendreau, Hertz, & Laporte, 1992); a tabu search stage that iterates over each customer and determines its best candidate neighbor solution until a minimum number of iterations are performed without improvement; and a final improvement stage. Dror & Trudeau (1989) presented a five–step heuristic algorithm for the VRPSD that first determines a good solution to the VRP and subsequently examines such a solution for potential split deliveries and selects the split with the best savings. Frizzell & Giffin (1995) published one construction and two improvement heuristics for the CVRP with time windows (each customer has one or more time intervals in which the delivery has to be made) and split deliveries (CVRPTWSD). They highlight that heuristics were initially thought to be adaptable to the split delivery problem by replacing a customer with demand q_i as q_i customers with unit demand. However, there would be a considerable increase in the number of customers. Mullaseril et al. (1997) provided an adaptation of the heuristic by Dror & Trudeau (1989) to solve a livestock feed distribution problem in a farm. Dror & Trudeau (1990) introduced the concept of split delivery routing and in a study that shed light on this important variant of the VRP through a series of experiments that contrasted split-delivery *vs.* traditional CVRP instances. Sierksma & Tijssen (1998) developed the Cluster–and–Route heuristic to schedule personnel pick–ups and drop–offs at offshore oil platforms. This heuristic first clusters the platforms and simultaneously determines their corresponding helicopter routes. This problem has the particularity that each person leaving the offshore platform would be replaced by a corresponding coworker. Thus, the number of personnel in the helicopter remained equal. Ho & Haugland (2004) authored a paper that solves the CVRPTWSD using a tabu search metaheuristic that first tries to determine a candidate feasible solution, then performs the tabu search, and finally executes a post–optimization procedure. Archetti et al. (2008) studied the VRPSD and determined, via an empirical investigation, that the more promising scenarios for the VRPSD to present higher savings are those in which the mean customer demand is slightly larger than half the vehicle capacity and those in which customer demand variance is small. They used *Granular Tabu Search* (Toth & Vigo, 2003) to generate the initial VRP solution and used the *SPLITABU* (C. Archetti et al., 2006) heuristic to derive the VRPSD solution. Archetti & Speranza (2012) published a survey on the VRPSD with its formulation, main properties, exact and heuristic solution methodologies developed thus far, and included an overview of published variations of the VRPSD such as time windows, pick-up and delivery, inventory and production, heterogeneous fleet, stochastic and discrete demands, among others.

The VRPSD has been studied from a multi-commodity perspective. (Claudia Archetti et al., 2014) published a study in which customers request multiple commodities and study effects of having vehicles that can carry all kinds of commodities versus vehicles that can only carry a set of products. However, commodities are assumed to have equal physical characteristics and vehicle capacities are treated as a numeric value. Berbotto et al. (2013) published Granular Tabu Search heuristic for the VRPSD. Their algorithm uses a randomized move at each neighborhood search and variant of the granularity threshold, determined for each tour. First, an initial solution is obtained via the savings algorithm (Clarke & Wright, 1964). Then, a Granular Tabu Search phase starts, which incorporates seven neighborhood search procedures, to finalize with a solution improvement stage. Tavakkoli-Moghaddam et al. (2007) present a metaheuristic designed to solve the VRPSD with a heterogeneous fleet. Their methodology involves the generation of an initial single-customer solution in which customers are allocated to available vehicles until their capacity is met or there are no customers left to assign. Then, a variant of Simulated Annealing is used to solve the split-delivery problem using a split-delivery oriented customization of the 1-opt and 2-opt moves. The results of SA to solve this CVRPSD were promising.

Proposed algorithm of simulated annealing

To date, tabu search has been a prevalent methodology for the solution of the VRPSD. It typically involves the creation of an initial feasible solution to the VRP (generally via heuristic methods) followed by exhaustive tabu searches and post-optimization procedures. However, simplistic approaches with comparable performance are valuable. The VRPSD literature has only marginally explored simulated annealing and thus the VRPC holds an opportunity to exploit such metaheuristic's strong search capabilities. The proposed simulated annealing-based metaheuristic algorithm for the VRPC is referred to as VRPCSA.

Problem definition

Let G = (V, E) be a complete graph, where $A = N \cup \{0\} = \{0,1,2,\dots,n\}$ is the set of nodes, and $E = \{(i,j) | (i,j) \in A \times A\}$ is the set of edges connecting each pair of nodes. Next, let $N = \{1,2,\dots,|N|\}$ be the set of customers, $|N| = n$, and $\{0\}$ the central depot. Each arc $(i,j) \in E$ has a Eucledian distance denoted by d_{ij}. Let $p \in P = \{1, 2, \dots, |P|\}$ represent the commodities required by customers, and $q_i^p = \{q_0^p, q_1^p, \dots, q_n^p\}$ for $i \in A, p \in P$ be the set of non-negative customer demands, while $q_0^p = 0 \; \forall \, p \in P$. Now, available vehicles $k \in K = \{1,2,\dots,|K|\}$ represent the fleet, where each k has a capacity of Q_k^f in terms of each facet $f \in F = \{1, \dots, |F|\}$. Facets correspond to any characteristic or dimension modeled in the problem (e.g. volume, weight, value). Each commodity p has a corresponding multiplier δ_p^f, which is a measure of the magnitude of each $p \in P$ for facet $f \, E \, F$. The objective is the minimization of the total distance traveled by the fleet, such that: (a) each route starts and ends at the depot; (b) each customer's demand must be exactly fulfilled; (c) the payload of any vehicle traveling through any arc (*i,j*) cannot be greater than Q_k^f for some facet *f*.

Description of LOADS model

This metaheuristic algorithm for the solution of the VRPC consists of a two-stage problem. On the one hand, routes are determined for each of the vehicles in the fleet using a simulated annealing-based metaheuristic. On the other hand, given a set of routes for the fleet (Obtained through the metaheuristic), payload must be assigned in an efficient way such that: (a) split deliveries are allowed, if there are any customers visited by more than one vehicle and if such alternative proves more viable (less costly); (b) shipment consolidation is permitted, as long as there are any split deliveries in the tours and commodity consolidation is less expensive; and (c) penalties due to over-capacitating the vehicles are avoided, where possible. The following linear programming model was developed to this end.

(11.1) $$\text{minimize} \sum_{(i,j)\in A} \sum_{k\in K} \sum_{p\in P} \rho_{ij}^{kp} \qquad \text{(LOADS)}$$

(11.2) $$\sum_{i,k\in\delta^-_{(m)}} u_{im}^{kp} - \sum_{j,k\in\delta^+_{(m)}} u_{mj}^{kp} = q_m^p \qquad \forall\, m \in N, p \in P$$

(11.3) $$\sum_{p\in P} u_{ij}^{kp} \cdot \delta_p^f - \frac{\rho_{ij}^{kp}}{\tau} \le Q_k^f \qquad \forall\, (i,j) \in A, k \in K, f \in F$$

(11.4) $$\sum_{i\in\delta^-_{(0)}} u_{i,\{0\}}^{kp} = 0 \qquad \forall\, \text{k} \in K, p \in P$$

(11.5) $$\rho_{ij}^{kp} \ge 0$$

(11.6) $$u_{ij}^{kp} \ge 0$$

(11.5) $$\sum_{i,k\in\delta^-_{(m)}} u_{im}^{kp} - \sum_{i,k\in\delta^+_{(m)}} u_{mj}^{kp} \ge 0 \qquad \forall\, m \in N, p \in P$$

The LOADS model minimizes all penalties ρ_{ij}^{kp} on vehicle *k*, of product *p,* over arc (*i,j*). Equations (11.2) are facsimile of (2.7), which enable commodity flow and demand fulfillment throughout the routes. The set of in-edges to *m* are denoted as $\delta_{(m)}^{-}$ while that of out-edges to *m* is written as $\delta_{(m)}^{+}$. Decision variable *u* in is a nonnegative decision variable that represents the unit load of commodity *p* in vehicle *k* over the arc (*i,j*). Essentially, the load of commodity *p* that arrives to customer *m*'s location on vehicle *k* has to: *1)* entirely (or partially) fulfill the *m^th^* customer's demand of said product (q_m^p), or depart in the same (or another) vehicle. Constraints (11.3) define the penalty incurred over each arc (*i, j*). τ is a constant and predefined penalty multiplier factor. Equations (11.4) guarantee that there is not commodity inflow back to the depot. Lastly, constraints (11.5) are added to the LOADS model to forbid shipment consolidation and have the model accept split deliveries while forbidding any mid-route commodity cross-docking. In the last step of the VRPCSA, integrality constraints are imposed on u_{ij}^{kp} to turn the LOADS model into a mixed-integer linear programming model. The objective is to have realistic loads returned by the model but only at the last stage, when a good-quality solution has likely been obtained. Prior to that stage, relaxing the integrality of u_{ij}^{kp} is a good compromise to reduce computational time and complexity of the LOADS model.

Algorithm definition

The proposed algorithm (VRPCSA) is based on the metal–merging metaheuristic and involves the following steps. **First**, the definition of a starting solution that consists of: the initialization of |*K*| empty tours, the random insertion of each $n \in N$ into a tour, regardless of its capacity utilization. Once all customers (nodes) have been assigned to a tour (vehicle), then then LOADS model is run, without integrality constraints on the vehicle payload. At this point, the starting solution has a set of tours that visit each customer once, the payload of each vehicle is determined for each edge, as well as any applicable penalties. This will be the starting point (Current solution). Then, a best-known solution is created as a clone of the current solution. **Second**, the core of the VRPCSA algorithm is executed. This step consists of a neighborhood search for each of a series of cycles during a temperature change. After the number of cycles per temperature is reached, the temperature *(T)* changes by a predetermined *cooling ratio (r),* and another series of cycles begins. During each cycle, a solution (c^*) which is a neighbor of the current solution is generated by performing a neighborhood search (See below). At this point, a sequence of visits for all tours is defined. Subsequently, the LOADS model is run, without integrality constraints on vehicle payload *u,* to determine payload allocation and, if beneficial, shipment consolidation. With the tours and payloads established, the total cost (Distance traveled plus penalty, if any) of the neighbor solution can be calculated. The *Metropolis algorithm* is used to determine whether the neighbor solution is accepted based on its total cost. If it is, it replaces the current solution. If the neighbor solution is feasible and its total cost is less than that of the best-known solution, the latter is replaced with the former. This process is repeated each cycle. During the first temperature cycle, the temperature must start at a high enough value such that, at least x_0 uphill moves (neighbor solutions with longer distances or higher penalties) are accepted. Otherwise, the temperature is increased (instead of decreased) until x_0 is satisfied[1]. This process is repeated until the termination criteria are met. These criteria involve either an algorithm total runtime limit or a predefined number of temperature changes without finding a best-known solution. **Third**, after the core portion of the VRPCSA is finished (termination criteria are met),

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[1] For this algorithm, the following parameter values were used: Cycles per temperature: 2500; Cooling ratio: .95; x_0 uphill moves: .85.

the LOADS model is executed one last time, with integrality constraints imposed on vehicle payloads *u*. A summary of this section is outlined on Algorithm 1.

Algorithm 1 - Pseudo code of the VRPCSA

```
VRPCSA
    Assign each i in N to initial solution X
    Set current solution c := X
    Set best solution b := X
    Set T := T0
    Do
        Set cycle := 0
        Do
            Generate c* (a neighbor of c)
            Run LOADS model for c*
            If f(c*) ≤ f(b) then b := c := c*
            Else ρ := f(c*) - f(c)
                If ρ < 0 then c := c*
                Else if rand() < e^(-ρ/T) then c := c*
            cycle ++
        While cycle < cycleLimit
        T := r * T
    While termination criteria are not met
End
```

Neighborhood search

Each temperature change has a series of temperature cycles. During each cycle, a randomly-selected neighborhood search is conducted. The following three neighborhood-generation mechanisms were developed for the VRPCSA. *i) Three-exchange:* a popular local search neighbor-generation mechanism, also known as *customer insertion.* This method consists of selecting a random customer and inserting it into a different position on the same or a different tour. Selection of which arcs to remove and which to add was done randomly, while forbidding the insertion of a customer into the same location from where is was originally removed. *ii) Split delivery insert:* since a split delivery is a key aspect of the mid-route shipment consolidation concept (Shipment consolidation can only occur if more than one vehicle visits the same customer), the evaluation of possible split deliveries across tours is paramount. Here, two main considerations were made: if the current solution was feasible, then a random customer from a random tour was selected; otherwise, then a random customer from the tour with the highest penalty was chosen as candidate for a split delivery. Then, a copy of the split-delivery candidate customer was inserted in a comparable way: if the current solution was feasible, then a random tour was selected; otherwise, a tour with the lowest penalty was chosen. Subsequently, a position in the chosen tour was randomly selected to insert the copy of the split-delivery candidate customer. *Split delivery delete:* since creating multiple split deliveries over possibly thousands of iterations could results in excessive duplication of nodes, a neighborhood search that removes split deliveries is essential. In this case, a random customer was selected, as long as said customer was visited by at least two vehicles, and removed from any of the possible tours by random selection.

Computational experiments

To corroborate the performance of the proposed algorithm to solve the VRPC, computational experiments are in order. Previous scholars have studied the VRPSD and concluded that the greatest benefits are only achieved under specific circumstances, particularly of customer demand (Claudia Archetti et al., 2008). Results point to the fact that there might exist certain conditions under which the models that use split deliveries perform better. This signals that the same might the case for the proposed VRPC model. The variables of interest of this experiment are outlined on Table 6.

The location of the depot is fixed across problems. Customer coordinates and customer demand are generated randomly. The fleet can have all vehicles of an equal capacity (Homogeneous) or it can have one vehicle with a larger capacity (Heterogeneous). If a customer's demand is negative, a transformation is used. Magnitudes δ_p^f (as presented on p. 29) of each commodity $p \in P = \{0,1\}$ are set to be different from one another, in terms of each facet $f \in F = \{0,1\}$, such that $2\delta_0^0 = \delta_0^1$ and $\delta_1^0 = 2\delta_1^1$. Vehicle capacities $Q_k^f = \{Q_k^0, Q_k^1\}$ were set to either {600, 600} or the necessary equal value to limit the number of standard vehicles to twenty. For the case of heterogeneous fleet, the capacity of large vehicles *k'* was $Q_{k'}^f = 2Q_k^f$. The experiment consists of a full factorial design comprising a set of 48 (3x2x2x2x2) different combinations. For each cell (Combination) in the factorial experimental design, ten problems were randomly generated and solved using the VRPC and VRPSD metaheuristic (With and without mid-route shipment consolidation, respectively). This brought the total number of different randomly generated problems to 480, and a total of 960 solved experiments. The VRPSD or, the non shipment-consolidating counterpart of the VRPCSA, works exactly like the VRPCSA except that in the former shipment consolidation is restricted at each node. Thus, the VRPSD can be used as a performance benchmark.

Table 6 - Experimental design for the VRPCSA metaheuristic

Variable		Levels	Values
Problem size		Small	300 customers
		Medium	400 customers
		Large	500 customers
Arc length variance		High	*μ = 1,000; σ = 50*
		Low	*μ = 1,000; σ = 1'000*
Fleet type		Homogeneous	Standard vehicles only
		Heterogeneous	Standard vehicles plus one large vehicle
Demand variance	Product 1	High	*μ = 100; σ = 20*
		Low	*μ = 100; σ = 200*
	Product 2	High	*μ = 100; σ = 20*
		Low	*μ = 100; σ = 200*

Results

Experiments were conducted in an Intel Core i7 machine with 8 GB of memory, running Ubuntu Linux 14 with Java (JRE and JDK) version 1.8 and IBM Ilog CPLEX version 12.7.1. Results were grouped by level of the network size (i.e. number of nodes) variable. Table 7, Table 8, and Table 9 summarize the results obtained for instances with 300, 400, and 500 nodes, respectively, across all combinations (cells) of the full-factorial experimental design. For all tables, *Fleet* = 1 when the generated problem had a heterogeneous fleet, homogeneous otherwise; a more negative gap implies better performance; and SD and XD is the number of nodes in which a split delivery and product consolidation takes place, respectively.

Table 7 - Performance gap of the VRPC vs. VRPSD metaheuristics for 300 customers

Cell	σ(arc length)	σ(demand P_1)	σ(demand P_2)	Fleet	Average SD	Average XD	Time (Min)	Gap Obj value
14	1000	20	200	1	13.60	13.10	3.64	-7.5%
13	1000	20	200	0	16.70	15.90	3.99	-6.9%
1	50	200	200	0	22.10	20.20	3.43	-6.7%
5	50	20	200	0	20.70	19.40	3.48	-6.4%
3	50	200	20	0	22.40	21.00	5.08	-5.8%
9	1000	200	200	0	15.70	14.40	2.38	-4.9%
2	50	200	200	1	18.00	17.00	5.68	-4.8%
11	1000	200	20	0	19.40	18.70	3.94	-4.7%
4	50	200	20	1	20.10	19.10	4.02	-3.2%
10	1000	200	200	1	14.90	14.50	3.51	-2.9%
6	50	20	200	1	22.40	21.30	4.63	-2.3%
8	50	20	20	1	14.80	13.30	3.12	-2.0%
7	50	20	20	0	12.60	12.10	3.37	-1.9%
15	1000	20	20	0	15.00	13.90	3.35	-1.9%
12	1000	200	20	1	19.80	18.00	3.49	-0.9%
16	1000	20	20	1	9.10	8.00	2.12	-0.1%

Table 8 - Performance gap of the VRPC vs. VRPSD metaheuristics for 400 customers

Cell	σ(arc length)	σ(demand P_1)	σ(demand P_2)	Fleet	Average SD	Average XD	Time (Min)	Gap Obj value
7	50	20	20	0	24.60	23.30	4.19	-10.2%
12	1000	200	20	1	17.70	17.30	3.78	-8.0%
5	50	20	200	0	22.80	21.40	5.45	-7.6%
1	50	200	200	0	23.30	22.10	4.50	-6.8%
9	1000	200	200	0	18.30	17.50	4.00	-5.8%
11	1000	200	20	0	18.00	17.20	4.40	-5.1%
2	50	200	200	1	20.10	19.20	5.59	-4.1%
4	50	200	20	1	20.80	20.30	4.85	-3.9%
10	1000	200	200	1	20.90	20.20	4.75	-3.9%
14	1000	20	200	1	19.70	18.90	4.05	-3.5%
13	1000	20	200	0	16.10	15.10	4.27	-2.7%
8	50	20	20	1	16.50	15.70	3.29	-2.4%
16	1000	20	20	1	18.80	17.60	3.28	-2.3%
15	1000	20	20	0	10.60	10.40	3.15	-1.7%
3	50	200	20	0	23.00	22.00	4.31	-1.3%
6	50	20	200	1	20.10	19.30	3.94	-1.1%

Table 9 - Performance gap of the VRPC vs. VRPSD metaheuristics for 500 customers

Cell	σ(arc length)	σ(demand P_1)	σ(demand P_2)	Fleet	Average SD	Average XD	Time (Min)	Gap Obj value
11	1000	200	20	0	19.00	18.70	4.47	-9.3%
1	50	200	200	0	24.40	23.50	6.52	-9.1%
9	1000	200	200	0	21.00	20.40	4.69	-8.9%
13	1000	20	200	0	17.70	17.10	4.17	-8.7%
10	1000	200	200	1	20.20	19.70	5.07	-5.9%
5	50	20	200	0	22.70	21.70	7.01	-5.2%
3	50	200	20	0	20.80	20.20	6.46	-4.3%
15	1000	20	20	0	18.70	17.40	5.34	-3.8%
12	1000	200	20	1	17.10	16.00	4.58	-3.7%
6	50	20	200	1	20.40	19.70	5.16	-3.1%
2	50	200	200	1	22.80	21.90	5.79	-2.2%
16	1000	20	20	1	16.60	15.80	4.81	-2.2%
7	50	20	20	0	17.10	16.50	4.21	-2.0%
4	50	200	20	1	20.90	20.20	5.12	-1.9%
8	50	20	20	1	17.10	16.60	4.52	-1.8%
14	1000	20	200	1	17.80	17.40	3.89	-0.5%

As expected, the benefits attained through mid-route shipment consolidation are sizeable, with better savings offered under some conditions. The results are sorted in descending order of the gap column, a comparison of the objective value obtained with the VRPCSA and the benchmark (Split deliveries allowed but no shipment consolidation). The first column (cell) indicates which combination of variables of interest was used for that instance (level at which each variable was set). The best-performing cases tend to revolve around the cells in which the variances of customer demands are high. Table 10 shows the average values obtained across all cells, grouped by problem size (i.e. number of nodes). Overall, it is clear that the VRPCSA can provide important savings while only performing shipment consolidation, on average on about five percent of the customer sites, with a comparable number of split deliveries. This signals that it is not required to perform consolidation in a large number of sites to get important savings. Furthermore, these savings can be larger, in cases in which customer demand is more volatile.

Table 10 - Average results of the VRPCSA by problem size

Problem size (|N|)	SD	XD	Time (Min)	Gap Obj value
300	17.33	16.24	3.70	-3.9%
400	19.46	18.59	4.24	-4.4%
500	19.64	18.93	5.11	-4.5%
Average	**18.81**	**17.92**	**4.35**	**-4.3%**

Conclusion

The performance of the proposed concept of shipment consolidation was tested under larger, more complicated problems. Given the NP-Hard nature inherent to the vehicle routing problem, coupled with the multi-commodity, heterogeneous fleet, characteristics of the VRPC, a metaheuristic was proposed to solve large problems under a reasonable amount of time. The metaheuristic is based on the popular metal-merging Simulated Annealing and is coupled with a linear programing model to distribute payload across vehicles. A set of neighborhood searches and various parameters for the calibration of the model were presented. Experimental results are showing important savings when compared to a non-shipment consolidating benchmark model, and instances with up to 500 nodes are solved within short computational times.

The proposed metaheuristic VRPCSA is a dual-stage algorithm that uses simulated annealing to define vehicles' routes, in tandem with an optimization algorithm to determine the optimal commodity allocation to available vehicles, given such fixed routes. This reduces the metaheuristic's runtime and complexity. A metaheuristic to determine not only the routes by also payload allocation to each arc, including possible mid-route shipment consolidation, could result in inferior performance or long computing times.

Future directions to investigate include the expansion of the possible neighborhood searches. To date, popular neighborhood search mechanisms include three-exchange (Relocation), four-exchange (Customer swap), and 2-Opt (Suzuki, 2016; Toth & Vigo, 2003; Xiao, Zhao, Kaku, & Xu, 2012). In this study, only the first of these popular searches was implemented to alter the sequence of customers in the tours. Possible extensions can also investigate the application of a dynamic neighborhood search that, instead of choosing a mechanism randomly, weighted probabilities with higher likelihoods of improving the solution are used. Furthermore, studies could extend the metaheuristic to see whether the use of parallel processing and multi-threading can be a way to evaluate multiple neighbors concurrently and to select the best move while avoiding dramatic impacts to computational time.

The contribution of this essay relies on the investigation of the benefits that the proposed concept of mid-route shipment consolidation can carry on large problem instances. As indicated by results of a computational experiment, consolidating shipment across vehicles as they complete their tours can be a powerful strategy to decrease distance covered by sizeable amounts. Furthermore, it appears payload consolidation need not be done multiple times across a single problem to get high savings. Consolidating shipments in as little as three percent of the nodes was enough to reach good-quality solutions. The proposed concept, coupled with the VRPCSA metaheuristic, can offer alternatives to logistics operations to improve performance while not requiring dramatic capital investments. The concept of shipment consolidation constitutes an interesting and novel alternative to vehicle routing operations.

CHAPTER 4: INCREASED PERFORMANCE THROUGH THE VEHICLE ROUTING PROBLEM WITH SHIPMENT CONSOLIDATION AND TIME WINDOWS

Introduction

The model for *vehicle routing problem with shipment consolidation* (VRPC) was shown to provide important benefits whilst requiring marginal efforts from carriers. It consists of an extended version of a vehicle routing problem with split deliveries which, in addition to multiple visits per customer, its vehicles can consolidate shipments mid-route. In other words, two or more vehicles can meet at a customer site and either partially or totally fulfill said customer's demand or rearrange shipments across vehicles. The improved total fleet capacity utilization results in better system-wide performance. However, the VRPC, like other variants of the Vehicle Routing Problem, initially focused on the minimization of total distance traveled. However, solely focusing on distance minimization can leave numerous factors underexplored. For example, delivery sequencing can dramatically impact average fuel expenditures (Suzuki, 2011; Xiao et al., 2012); driving patterns significantly affect the vehicle's efficiency (Ericsson, 2001); and road gradient, selection of tires, engine power (Coyle, 2007) can affect performance to a certain degree. These are only a few relevant factors that might inflict substantive change in a fleet's routing policy when properly accounted for. The objective of the proposed model for the *Vehicle Routing Problem with Shipment Consolidation and Time Windows* (VRPCTW) is to shed light on some of the factors that might affect the decision to consolidate shipments mid-route. To this end, sequential interviews with supply chain logistics practitioners were conducted to outline some of the most delicate shortcomings of a theoretical VRPC. The interviews consisted of interactions with large companies that operate in the TL and LTL segments of transportation. Two of the companies that provided insight have a private fleet of vehicle while a third focuses mainly on TL services but offers LTL through partners. The results suggested that some of the most important aspects that required further investigation were: *i) time scheduling:* while mid-route shipment consolidation may be proven useful, it is also true that time windows is a key aspect of vehicle coordination. Especially since vehicles are expected to meet at various customer sites, extended waiting times for package consolidation could result in prohibitively expensive operations; *ii) customer site feasibility:* a problem with mid-route package consolidation among vehicles is that such operations may require specialized equipment and facilities. One of the interviewed companies expressed high concerns over making deliveries to a customer whose facilities have restricted space availability. The scenario wherein two vehicles use its location to perform any shipment consolidation seemed improbable. For this reason, this study also considers the impact of some customer sites not having enough spatial resources to accommodate any more than one vehicle at a time; and *iii) added labor cost:* mid-route consolidation of packages can significantly decrease total distance traveled by the fleet. However, performing package consolidation at customer sites requires extra t. In some cases, the time spent doing consolidation of any kind may slow operations to the point any consolidation efforts may prove futile. This essay analyzes the impact of any additional times required to serve the customers in the network. The proposed model has a new time dimension, which exponentially increases the complexity of the resulting model per the number of customers.

This study comes at time wherein business logistics costs in the U.S., which have grown at an average 4.6 percent from 2010 to 2014, while transportation is 5.5 percent its average annual component. The logistics industry is expected to show higher transportation rates in 2017 and, though the decrease in price of crude oil did slow the growth of transportation costs, it remains important as oil price risks will contribute to expected surcharges in transportation costs.

The fact remains that the transportation sector is undergoing important changes with an average growth of 7.5 percent between parcel and the LTL segments for 2015. If business logistics continues to conform a considerable segment of the U.S. nominal GPD at around 7.9 percent (ATKearney & Penske, 2016), the calls for rigorous research are strongly warranted as the logistics industry continues to become stronger and is pushed for further innovation. The proposed model for the *Vehicle Routing Problem with Shipment Consolidation and Time Windows* (VRPCTW) not only accounts for aspects regarded as highly impactful per industry recommendations (i.e. time scheduling, customer site feasibility, added labor cost), but also builds on a new vehicle routing problem previously unstudied in the academic literature, thus pushing the boundaries of the available epistemology of the transportation field within business logistics research.

Literature Review

The proposed model for the *Vehicle Routing Problem with Shipment Consolidation and Time Windows* (VRPCTW) is a specialized form of the vehicle routing problem with split deliveries, time windows, a fleet with potentially heterogeneous characteristics, multiple commodities, multiple dimensions for the demand, package consolidation support at customer sites, and an assessment of the impact of any additional labor cost. However, the VRPC is a newly proposed model, to the best of the author's knowledge. We will discuss some of basic forms of other comparable models upon which the VRPCTW is built.

The vehicle routing problems with time windows is an extension of the traditional capacitated vehicle routing problem with the restriction that each customer declares a specific window of time during which the shipment must be delivered. There are two types of time windows: hard and soft. The former refers to the case in which the service (unloading) cannot be performed outside of the time window. The latter, is when servicing a customer outside of its time window is permitted but at the expense of a penalty (Toth & Vigo, 2002b). Typically, the objective is to minimize total distance traveled. However, some models assume an infinitely-sized fleet and add a term to the objective function to induce the minimization of the number vehicles routes, typically done by adding a fixed cost to dispatching one vehicle. The vehicle routing problem with time constraints is considered more difficult than its not time-constrained counterpart (Kolen et al., 1987) and since the VRP is considered NP-hard (Solomon & Desrosiers, 1988) then, by transitivity, the VRPTW is also NP-hard. Initial works on the VRP with time windows date back to the late 80s. Most of the literature assumes a heterogeneous fleet but extensions of previous publications to account for multiple types of vehicles is possible. Typically, instances in the literature have had varying number of customers, with a set of 100-customer instances presented by Solomon (1987) which gained respectable popularity in the field. Exact algorithms (Baldacci, Mingozzi, & Roberti, 2012; Kallehauge, 2008) and heuristics (Braysy & Gendreau, 2005; Kallehauge, Larsen, Madsen, & Solomon, 2005) were initially developed for the VRPTW but metaheuristics, especially tabu search and genetic algorithms, closely followed the scholarly progression of this sub-stream of literature. (Michel Gendreau & Tarantilis, 2010; Toth & Vigo, 2014).

Proposed model

The proposed model for vehicle routing with shipment consolidation and time windows aims to unfold the benefits of shipment consolidation under more complicated scenarios. Earlier versions of the proposed model were presented to and discussed with representatives of large companies based in the U.S. and that operate in the LTL and TL sectors of transportation. Their input was particularly useful to determine crucial factors that could limit the implementation of shipment consolidation. As mentioned above, time coordination is of paramount importance, as are prohibitively expensive idling times at customer locations. The proposed model of the *Vehicle Routing Problem with Shipment Consolidation and Time Windows* (VRPCTW) addresses these concerns. Another important aspect of the concept of mid-route shipment consolidation is the need for extra resources such as space or specialized equipment. Two of the interviewed firms stated their concern as to some customers' inability to support consolidation, given that two trucks would have to occupy their bays momentarily. Thus, the VRPCTW includes a restriction for customers' facilities that cannot support payload consolidation.

The VRPCTW is based on traditional *source-to-sink* routing models with time windows and is defined by digraph G = (V, E), where $A = \{0\} \cup N \cup \{n+1\} = \{0, 1, 2, \ldots, n, n+1\}$ is the full set of nodes and $E = \{(i,j) | (i,j) \in A \times A\}$ is the set of edges joining each pair of nodes. The set of customers is $N = \{1,2,\ldots,|N|\} \setminus \{0, n+1\}$ and $|N| = n$. Elements $\{0\}$ and $\{n+1\}$ correspond to the source and sink depot, respectively. The *sink* acts as a virtual copy of the *source* depot, wherein vehicles arrive after completion of their tour. The distance of each arc $(i,j) \in E$ is denoted by d_{ij}, while the time it takes to traverse it t_{ij}. Let $p \in P = \{1, 2, \ldots, |P|\}$ represent the various commodities, while $q_i^p = \{q_0^p, q_1^p, \ldots, q_n^p\}$ for $i \in A, p \in P$ be the set of non-negative customer demands, with $q_0^p = 0 \; \forall \, p \in P$ and $q_0^p = q_{n+1}^p$. The fleet is comprised of vehicle $k \in K = \{1,2,\ldots,|K|\}$, in which each vehicle k is limited by a capacity of Q_k^f for each facet $f \in F = \{1, \ldots, |F|\}$. Each commodity p has a corresponding multiplier δ_p^f, which is a measure of the magnitude of each $p \in P$ for a facet $f \in F$. In addition, each node $i \in A$ holds a time window $[a_i, b_i]$ during which it must be served. Early arrivals are allowed at the cost of the carrier but no late deliveries are possible. Each vehicle must arrive to each customer's location with enough time (s_i) to completely unload each unit of the shipment delivered to i before the expiration of the time window. Mid-route shipment consolidation implies that payload can be transferred across vehicles at any customer's location. The proposed model is as follows.

$$\text{minimize} \sum_{\substack{(i,j)\in A \\ k\in K}} t_{ij}\cdot x_{ij}^{k} + \sum_{\substack{i\in N \\ k\in K}} w_{ik} + \sum_{\substack{i\in A \\ k\in K \\ p\in P}} s_i\left(y_i^{kp+} + y_i^{kp-}\right) \tag{12.1}$$ (VRPCTW)

subject to

$$\sum_{i\in A, k\in K} x_{ij}^{k} \geq 1 \qquad \forall\, j\in N \tag{12.2}$$

$$\sum_{j\in A} x_{0j}^{k} \leq 1 \qquad \forall\, k\in K \tag{12.3}$$

$$\sum_{i\in A} x_{im}^{k} - \sum_{j\in A} x_{mj}^{k} = 0 \qquad \forall\, m\in N, k\in K \tag{12.4}$$

$$\sum_{i\in N\cup\{0\}} x_{i,n+1}^{k} = 1 \qquad \forall\, k\in K \tag{12.5}$$

$$x_{ii}^{k} = 0 \qquad \forall\, i\in A, k\in K \tag{12.6}$$

$$\sum_{(i,j)\in S} x_{ij}^{k} \leq |S|-1 \qquad \forall\, S\subseteq N, S\neq\emptyset, k\in K \tag{12.7}$$

$$y_i^{kp+} \geq \sum_{m\in A} u_{mi}^{kp} - \sum_{m\in A} u_{im}^{kp} \qquad \forall\, i\in A, p\in P, k\in K \tag{12.8a}$$

$$y_i^{kp-} \geq \sum_{m\in A} u_{im}^{kp} - \sum_{m\in A} u_{mi}^{kp} \qquad \forall\, i\in A, p\in P, k\in K \tag{12.8b}$$

$$\phi_i^{kp} \leq M\cdot y_i^{kp-} \qquad \forall\, i\in N, k\in K, p\in P \tag{12.9a}$$

$$\phi_i^{kp} \geq {}^{1}\!/\!_{M}\cdot y_i^{kp-} \qquad \forall\, i\in N, k\in K, p\in P \tag{12.9b}$$

$$o_i^{k} + \sum_{p\in P} s_i\left(y_i^{kp+} + y_i^{kp-}\right) + t_{ij} \leq T_j^{k} + (1-x_{ij}^{k})\cdot M \tag{12.10a}$$

$$\forall\, i\in N\cup\{0\}, j\in N\cup\{n+1\}, i\neq j, k\in K$$

(12.10b) $$T_j^k + (x_{ij}^k - 1) \cdot M \le o_i^k + \sum_{p \in P} s_i \left(y_i^{kp+} + y_i^{kp-} \right) + t_{ij}$$

$$\forall\, i \in N \cup \{0\},\ j \in N \cup \{n+1\}, i \ne j, k \in K$$

(12.10c) $$o_i^k \ge o_i^{k'} - M(1 - \phi_i^{kp}) \qquad \forall\, i \in N, p \in P, (k,k') \in K \mid k \ne k'$$

(12.11a) $$o_i^k \ge T_i^k \qquad \forall\, i \in A, k \in K$$

(12.11b) $$o_i^k \ge a_i \cdot \left(\sum_{m \in A} x_{mi}^k \right) \qquad \forall\, i \in A, k \in K$$

(12.12) $$w_i^k = o_i^k - T_i^k \qquad \forall\, i \in A, k \in K$$

(12.13) $$o_i^k + \sum_{p \in P} s_i \cdot y_i^{kp+} \le b_i \cdot \left(\sum_{m \in A} x_{mi}^k \right) \qquad \forall\, i \in N, k \in K$$

(12.14) $$\sum_{i \in A, p \in P, k \in K} u_{i,n+1}^{kp} = 0$$

(12.15) $$\sum_{k \in K} y_{n+1}^{kp+} = 0$$

(12.16) $$\sum_{i \in A, k \in K} u_{im}^{kp} - \sum_{j \in A, k \in K} u_{mj}^{kp} = q_m^p \qquad \forall\, m \in N, p \in P$$

(12.17) $$u_{ij}^{kp} \le M \cdot x_{ij}^k \qquad \forall\, (i,j) \in A, k \in K, p \in P$$

(12.18) $$\sum_{p \in P} u_{ij}^{kp} \cdot \delta_p^f \le Q_k^f \qquad \forall\, (i,j) \in A, k \in K, f \in F$$

(12.19) $$\sum_{i \in A} u_{im}^{kp} - \sum_{j \in A} u_{mj}^{kp} \ge 0 \qquad \forall\, m \mid A_m = 0, p \in P, k \in K$$

(12.20) $$w_{\{n+1\}}^k = 0 \qquad \forall\, k \in K$$

(12.21) $$T_0^k = 0 \qquad \forall\, k \in K$$

(12.22) $$\phi_0^{kp} + \phi_{n+1}^{kp} = 0 \qquad \forall\, p \in P, k \in K$$

(12.23) $$x_{ij}^k \in \{0,1\}$$

(12.24) $$w_i^k \geq 0$$

(12.25) $$y_i^{kp+} \geq 0$$

(12.26) $$y_i^{kp-} \geq 0$$

(12.27) $$\phi_i^{kp} \in \{0,1\}$$

(12.28) $$u_{ij}^{kp} \in \mathrm{N}^0 = \{0,1,2,\ldots\}$$

(12.29) $$o_i^k \geq 0$$

(12.30) $$T_i^k \geq 0$$

(13) $$[a_0, b_0] = [a_{n+1}, b_{n+1}]$$

(14) $$a_0 \leq \min_{i \in A \setminus \{0\}} \{b_i - t_{0i}\} = a_{n+1}$$

(15) $$b_0 \geq \min_{i \in A \setminus \{0\}} \{max\{a_0 + t_{0i}, a_i\} + S_i + t_{i,n+1}\}$$

(16) $$c_{0,n+1} = t_{0,n+1} = c_{n+1,0} = t_{n+1,0} = 0$$

Objective function (12.1) minimizes total time to serve the system represented by: that spent traveling the necessary arcs; waiting time, if any, at customer sites; and time spent in material handling, which includes shipment consolidation, if any. To obtain t_{ij}, d_{ij} is divided by a predetermined factor of speed. x_{ij}^k is a binary decision variable, which equals *one* if vehicle k travels arc (i,j), *zero* otherwise. w_i^k represents vehicle k's waiting time at customer site i before unloading begins. Finally, y_i^{kp+} and y_i^{kp-} represent the number of units of product p being unloaded from and loaded onto vehicle k at customer i, respectively; while s_i is the expected unit product handling time at customer i. Based on repeated interactions with various companies operating in the TL and LTL segment of transportation, a pressing concern of implementing the proposed VRPCTW is that any savings would be heavily outweighed by the necessary additional time that mid-route shipment consolidation events would add to the fleet's routes. Thus, time spent serving the system can potentially be the most important variable to assess the viability of the VRPCTW model.

The set of constraints (12.2) warrant for at least one visit to each customer. (12.3) seek that vehicles depart the depot at most once. These constraints have the embedded objective of identifying whether there is any excess capacity in the form of unused vehicles. (12.4) correspond to network flow constraints. These ensure that the number of vehicles arriving to each customer equals the number of vehicles departing from it. Constraints (12.5) model the arrival of all vehicles to *sink* $\{n+1\}$. The set of constraints (12.6) prevent any recurrent arcs. (12.7) are the traditional *subtour elimination constraints,* modeled via the powerset S of A, wherein the cardinality of S is used to determine the maximum number of arcs in a tour. Next, (12.8a) and (12.8b) model the inflow (y_i^{kp-}) and outflow (y_i^{kp+}) of units of product p from or onto vehicle k at each location i, as stated above. Subsequently, (12.9a) and (12.9b) correspond to the definition of ϕ_i^{kp}, a binary decision variable equal to one when product p is being consolidated on vehicle k at site i; zero otherwise.

Naturally, the next constraints (12.10a), (12.10b), and (12.10c) ensure proper time flow throughout the routes of each vehicle. Two non-negative decision variables T_i^k and o_i^k represent the time when vehicle k arrives to customer i and the time in which service begins at customer i from vehicle k, respectively. Constraints (12.10c) use indicator variable ϕ_i^{kp} to force vehicles being resupplied with product to wait for any other vehicles to arrive to the customer's site. Pertaining the time windows, (12.11a) determines that o_i^k never occurs before T_i^k; while (12.11b) seeks that service time does not begin before the beginning (a_i) of customer i's time window [a_i, b_i]. Furthermore, (12.12) define w_i^k or, vehicle *k*'s waiting time at customer *i*'s location. Next, (12.13) ensures that the service must be finished by the end (b_i) of customer *i*'s time window. The set of constraints (12.14) eliminates any inflow of product to the depot (sink) through *u,* a nonnegative, integer-valued decision variable that symbolizes the unit load of product *p* in vehicle *k* over the arc (*i,j*). Constraints (12.15) prevent material handling and unnecessary payload upon arrival back to the depot. (12.16) enable mid-route commodity consolidation by seeking product flow and demand fulfillment throughout the network. In short, product *p* that arrives to customer *m*'s location on vehicle *k* has to: *1)* entirely (or partially) fulfill the m^{th} customer's demand of such product (q_m^p), or depart in the same (or a different) vehicle. In (12.17) decision variables *u* and *x* are connected such that if a vehicle *k* is not traveling through arc (*i, j*) then its load *u* on that same arc should be zero. Constraints (12.18) are also known as capacity cut constraints, forbidding the load of any vehicle *k* traversing the arc (*i,j*) to exceed *k'*s capacity in terms of a certain facet *f* at any point[2].

Interactions with several industry practitioners raised a concern about how, due to various limitations, not all customers may allow shipment consolidation among multiple vehicles at their facilities. Thus constraints (12.19) take part in restricting consolidation only to customer sites that can support it. There, indicator variable $A_m = 1$ when customer *m* has the resources, capability, and is willing to allow consolidation on their premises; zero otherwise. (12.20) and (12.21) determine that there should not be any waiting time at the depot (sink) and all vehicles

[2] It may seem that a solution could become infeasible if the same vehicle traveled the same arc twice at a different time. However, even though it may seem theoretically possible, this behavior should not occur in optimal solutions to the proposed model. For vehicle *k* to travel the arc (*i, j*) twice, *k* would have to make at least two visits to each of *i* and *j* with commodity consolidation at some node $m \in N\backslash\{0, n+1\}$. Since this model assumes the triangle inequality holds for the C_{ij} and t_{ij} matrices, visiting both *i* and *j* twice would add unnecessary travel time/distance and an alternative solution, with lower objective value, could be easily found. To corroborate this an empirical analysis of select solutions in the present study was done, wherein no optimal solution involved the same vehicle making more than one visit to a specific customer.

start their tours at the depot (source), respectively. (12.22) set conditions for ϕ_i^{kp} at the depots and are set at zero, since shipment consolidation is a concept that only applies to customer sites and is thus applicable to neither the source or the sink. The sets (12.23) through (12.30) correspond to the decision variables in the VRPCTW. (13) determines the time windows for the source and sink depots, (14) and (15) determine essential conditions of such time windows, and (16) fix the cost *c* and time *t* between the arcs from the source to the sink.

Relevance of the VRPCTW

Analysis of the performance of the VRPC showed capabilities congruent with significant improvements in routing operations. The VRPC model expands the solution domain to improve the way in which trucks are routed across the network and allows realistic routine operations such as shipment consolidation. However, the VRPC, though theoretically advantageous, has certain pitfalls that caused practitioners to question its business value. Some of the shortcomings foreseen by the interviewed supply chain professionals were the lack of schedule coordination, customer site viability for mid-route consolidation of shipments, and possible additional labor costs. In response, the VRPCTW answers a call for further, deeper studying of the concept of mid-route shipment consolidation. But the relevance of the VRPCTW is not only business driven. The U.S. government has approved funds in the amount of more than 300 billion to improve logistic infrastructure through 2020. Pertinently, almost three quarters of this amount is destined for Federal Highway Administration. Thus, the ground logistics segment is notably receiving important attention. Programs as such have partially been influenced by the fact that the parcel and express sector in the U.S. has exhibited a continuous growth and currently represents around 46 percent of all domestic shipments. This trend is in part explained by the value customers are placing in e-commerce and omnichannel trends. The more realistic variables incorporated in the VRPCTW model, along with flexibility to add virtually unlimited commodities and all possible dimensions (facets) of interest, make this model an appealing routing solution to motors carriers.

Computational experiments

To study the performance of the VRPCTW, the following experimental design was developed to obtain a better assessment of the performance of the VRPCTW under assumptions that would more suitably resemble a firm's operations. The variables of interest, along with their corresponding levels, are outlined in Table 11, next. As stated above, δ_p^f were set such that commodities $p \in P = \{0,1\}$ were different from one another, for each facet $f \in F = \{0,1\}$. Thus, $2\delta_0^0 = \delta_0^1$ and $\delta_1^0 = 2\delta_1^1$. Vehicle capacities $Q_k^f = \{Q_k^0, Q_k^1\}$ were set to {600, 600} for standard vehicles, while for large vehicles *k'* the capacity was $Q_{k'}^f = 2Q_k^f \; \forall f \in F$.

The arc length variance varies the geographical location of the customers. With a level of high, customers tend to be more scattered throughout the network, while with a low arc variance, they tend to appear more closely grouped. Fleet type consists of using a single type of vehicle with a standard capacity (homogeneous) as opposed to using a fleet comprised of all standard vehicles except one (heterogeneous), with twice the capacity. Demand variance varies across two levels (high *vs.* low) to determine whether customer demand may impact any benefits obtained through the VRPCTW model.

In the proposed model of VRPCTW, the customer site consolidation capability $A_m = 1$ when customer *m* has the resources and willingness to support consolidation of the carrier's shipments at its facilities, zero otherwise. Finally, the width of the time window is also varied across two levels (high vs. low). Expected performance is such that the narrower the time window the smaller the number of feasible solutions that will exist. Several preliminary rounds of this experiment were conducted in an effort to estimate a cap for the cardinality of N, such that each problem was solved to optimality within a reasonable amount of time. After careful adjustments of several optimization parameters and some improvements in the algorithm design and the implemented code, only small instances were solved within reasonable time frames. Any instance with eight nodes spent at least 24 hours without finding an optimal solution. Feasible solutions were often found quickly, but an optimal solution generally took far longer. In the interest of preserving computing times at reasonable levels, the network size was set at two levels: 5 and 6.

Table 11 - Experimental Design for the VRPCTW

Variable		Levels	Values
Arc length variance		High	μ = 1,000; σ = 50
		Low	μ = 1,000; σ = 1,000
Fleet type		Homogeneous	Standard vehicles only
		Heterogeneous	Standard vehicles plus one large vehicle
Demand variance	Product 1	High	μ = 100; σ = 200
		Low	μ = 100; σ = 20
	Product 2	High	μ = 100; σ = 200
		Low	μ = 100; σ = 20
Customer site consolidation capability (A_m)		High	$A_m = 1 \ \forall m \in N$
		Medium	A_m = 0 for 1/3 of the customers
		Low	A_m = 0 for 2/3 of the customers
Time window width		Low	width = 50
		High	width = 100

The experimental design consists of a total of 96 combinations (cells) from six factors (2 x 2 x 2 x 2 x 3 x 2) studied. For each cell, 15 problems were randomly created, for a total number of 1440 generated problems. To present a comparison, each of these problems was solved with both the VRPCTW and the VRPSDTW. The latter, is essentially the same as the VRPCTW but with the restriction that shipment consolidation is not permitted ($A_m = 0 \; \forall \; m \in N$). This design raised the total number of problems solved to 2880. In addition, two network sizes were employed (5-node and 6-node instances) thus doubling the total number of generated problems to 2880, with a final dataset size of 5760 solved instances. Each problem was generated in Java (JDK and JRE 1.8) and solved using IBM Ilog CPLEX version 12.6.3. The experiments were conducted in a quad-core Intel Core i7 machine with 16 GB of memory running Windows 10.

Results

A performance benchmark was established by comparing the two models VRPCTW and the VPRSDTW (Both capacitated vehicle routing models that allow split deliveries but only the former permits shipment consolidation at customer sites). The top twelve best-performing experimental units (cells) are reported in Table 12 and Table 13 for problems with 5 and 6 nodes, respectively. The full report is included in the Appendix (Table 15 and Table 16).

Table 12 - Top twelve *best*-performing cells of the VRPCTW *vs*. VRPSDTW with 5 nodes

Cell	σ(arc length)	σ(demand P_1)	σ(demand P_2)	Fleet	Am Percent	tw Width	Gap Obj value
1	50	200	200	0	0.667	50	-2.9%
57	1000	200	200	1	0.333	50	-2.0%
52	1000	200	200	0	0.333	100	-1.6%
71	1000	200	20	1	0	50	-1.2%
4	50	200	200	0	0.333	100	-0.6%
49	1000	200	200	0	0.667	50	-0.5%
56	1000	200	200	1	0.667	100	-0.4%
90	1000	20	20	0	0	100	-0.4%
69	1000	200	20	1	0.333	50	-0.4%
3	50	200	200	0	0.333	50	-0.2%
84	1000	20	200	1	0	100	-0.2%
12	50	200	200	1	0	100	-0.1%

Notes: Fleet is equal to one when the generated problem had a heterogeneous fleet, homogeneous otherwise
A more negative gap implies better performance

Table 13 - Top twelve *best*-performing cells of the VRPCTW *vs*. VRPSDTW with 6 nodes

Cell	σ(arc length)	σ(demand P_1)	σ(demand P_2)	Fleet	Am Percent	tw Width	Gap Obj value
42	50	20	20	0	0	100	-10.5%
3	50	200	200	0	0.333	50	-4.8%
41	50	20	20	0	0	50	-4.4%
18	50	200	20	0	0	100	-3.9%
89	1000	20	20	0	0	50	-2.7%
59	1000	200	200	1	0	50	-2.6%
90	1000	20	20	0	0	100	-2.5%
17	50	200	20	0	0	50	-2.4%
77	1000	20	200	0	0	50	-2.2%
72	1000	200	20	1	0	100	-1.7%
85	1000	20	20	0	0.667	50	-1.6%
9	50	200	200	1	0.333	50	-1.5%

Notes: Fleet is equal to one when the generated problem had a heterogeneous fleet, homogeneous otherwise
A more negative gap implies better performance

Results suggest an important advantage than can be attained, even at low network sizes, through the implementation of the VRPCTW. Total time spent both traveling the network while fulfilling customer demand, including waiting time, if any, and shipment consolidation time can reach improvements in magnitudes higher than ten percent on average (across the 15 instances solved for each experimental unit) with the VRPCTW *vis-à-vis* the VRPSDTW. However, given that not all cells provided a lower average objective function value, the VRPCTW may have better performance only on some network typologies. Consolidation was performed only on 93 of 2880 generated instances (3.2%) but their savings (across all experimental units) in objective function value were on average 6.5 and 11.4 percent for the 5 and 6 nodes, respectively; with a grand average of 10.1 percent savings across all instances with shipment consolidation. Comparatively, average vehicle waiting time at customer sites decreased by 17.2 percent across all instances in the dataset with shipment consolidation.

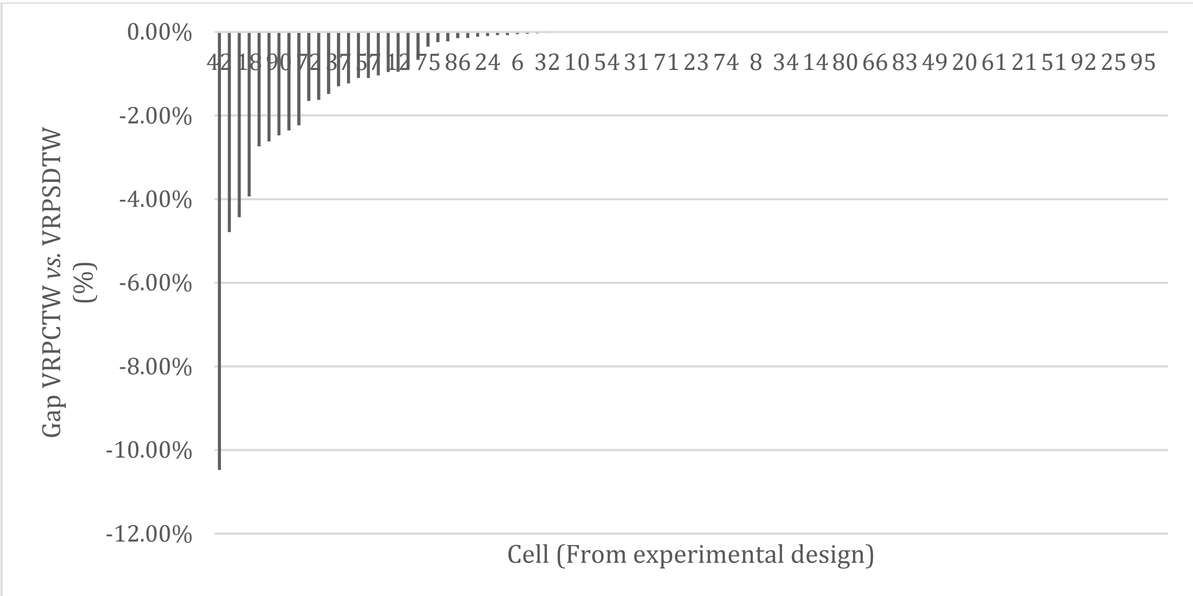

Figure 14 - Average performance gap of the objective value of the VRPCTW *vs.* VRPSDTW with the six-node problems

The average performance gap between the VRPCTW and the VRPSDTW was calculated for each cell in the full-factorial experimental design, using the 6-node problems. As depicted on Figure 14, some cells resulted in savings thanks to shipment consolidation with the VRPCTW. However, the remaining cells did not show any positive improvement versus the benchmark (VRPSDTW). To investigate further, a regression subset selection model, by exhaustive search, was employed using the six variables manipulated in the experiment. The leaps model was used with Mallow's C_p to asses fit. As displayed on Figure 15, the best predictors for the gap (Improvement of the VRPCTW vs. VRPSDTW) in the objective function value (Excluding the intercept) were: variance of the arc length (set at high), product 2's demand variance (set at high), and time windows width (Set at high). Second-order effects show significant interactions of: variance of the arc length (high) with variance of P_2's demand (high); variance of the arch length (high) with Customer site consolidation capability (A_m) set to medium; variance of the arch length (high) with time window width (high); variance of P_1's demand (high) with fleet (Heterogenous); variance of P_2's demand (high) with time window width (high); variance of

P_2's demand (high) with Customer site consolidation capability (A_m) set to medium; and fleet (Heterogenenous) with Customer site consolidation capability (A_m) set to medium. These results indicate that it may be better to implement a mid-route shipment consolidation strategy when the network has a generally high variance of aspects such as customer location, demand patters, coupled with a heterogeneous fleet, in some cases. Should these conditions be met, system performance can be expected to increase.

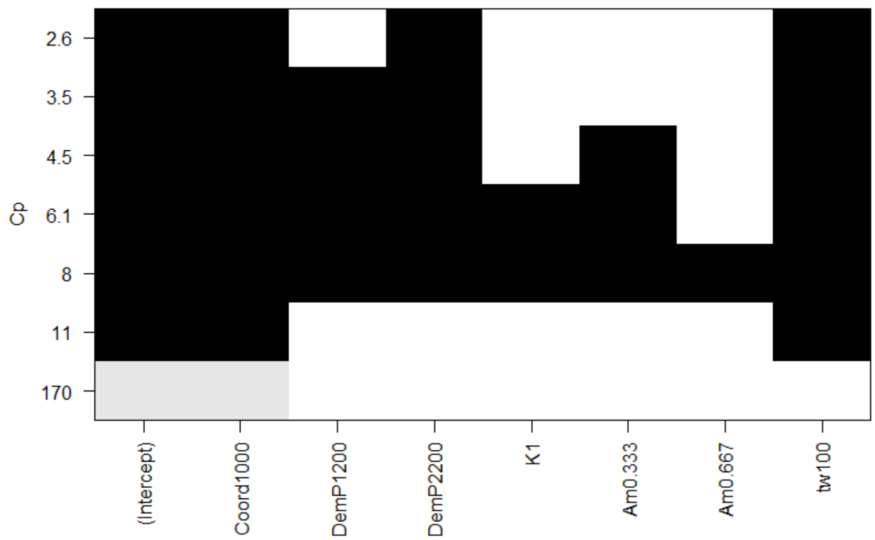

Figure 15 - Leaps variable selection plot

Discussion

Results from Chapter 1 suggest the cases in which the VRPC is more likely to perform better are those in which the variance of the arc length and the demand of products 1 and 2 high, and if the fleet is heterogeneous. The results from the VRPCTW are similar in that, for this model, the best predictors of performance gap were arc length and product variance, along with time window width. One important concern of the practitioners who provided feedback on earlier versions of the proposed concept (mid-route shipment consolidation) was the impact that customer site consolidation capability could have on the utility of the VRPCTW. However, results show that this should not be too worrisome. Figure 16 shows the two network sizes studied and depicts the average performance gap (More negative Gaps are better) of the VRPCTW *vs.* VRPSDTW in terms of Customer Consolidation Capability (A_m) of the instances in which shipment consolidation took place[3]. The graphic shows savings at various levels of A_m. Furthermore, results are surprising in that, intuitively, the fewer the customers sites that support consolidation, the smaller the expected savings. This surprising finding calls for further research into the particularities that drive better performance through shipment consolidation such as characteristics of the customers' positions in the network or demand patterns of some customers relative to others, etc.

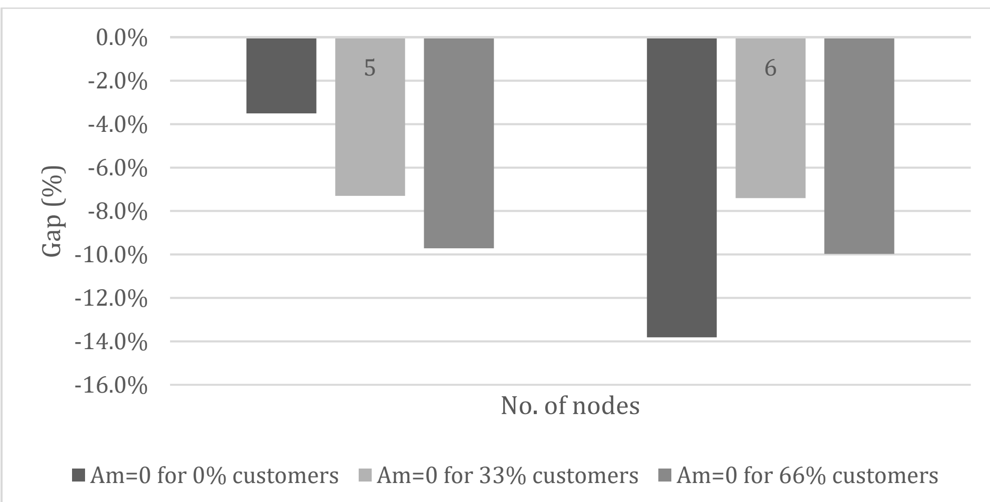

Figure 16 - Performance of the VRPCTW vs. VRPSDTW by levels of customer consolidation capability (Am) and network size for instances with shipment consolidation

One important aspect to note, regarding the performance of the VRPCTW, is that it appears savings will not necessarily increase if more shipment consolidation is done throughout the system. First, Figure 17 shows how savings diminish (More negative Gaps imply better performance) as the number of consolidation points increases with the VRPCTW. Furthermore,

[3] The instances in which shipment consolidation did not take place had equal performance to the benchmark models, which allow traditional split deliveries but no reloading of payload onto vehicles.

the largest number of consolidation sites is not any higher than two. This indicates that, though shipment consolidation can carry important savings, it is not necessarily better to implement it multiple times throughout the routes. The trade-off is that at some point, the time required to consolidate shipments may begin to outweigh any savings consolidating can offer. Thus, fleet routes can be established with only a limited number of consolidation points in mind, while attaining good savings.

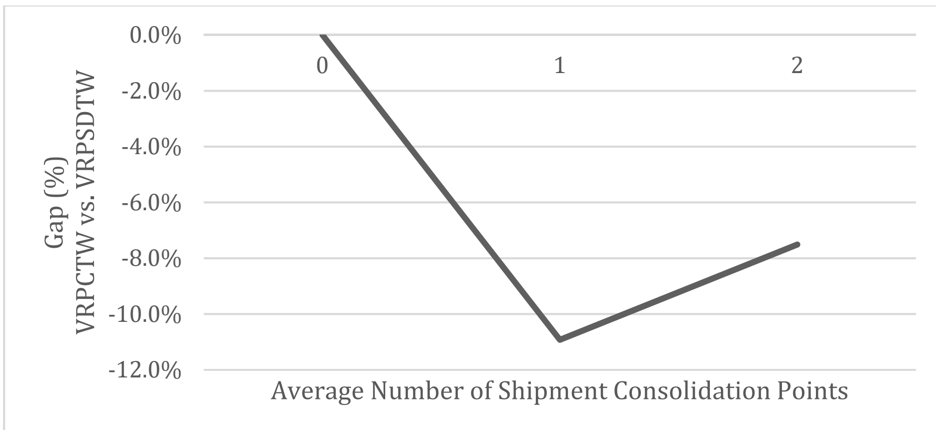

Figure 17 - Optimality gap of the VRPCTW *vs.* VRPSDTW by number of shipment consolidation points for *all* cells

Conclusion

The *Vehicle Routing Problem with Shipment Consolidation and Time Windows* (VRPCTW) was presented in this essay as a response to calls for further inquiry of the proposed model of mid-route shipment consolidation. Multiple interviews were conducted with representatives from important transportation logistics companies operating in the TL and LTL sectors. Through these interactions with industry, the VRPCTW was refined and relevant variables were studied, such as time coordination, capability of customer sites to support shipment consolidation, time commitment, and varying fleet capacity. An experimental design consisting of a total six variables was designed and nearly three thousand different problems were randomly generated and solved to optimality with and without shipment consolidation.

Results show that the VRPCTW can offer sizeable savings, even on small network sizes. The VRPCTW did not provide savings in all cells of the experimental design. However, the worst objective function values obtained with the VRPCTW are at least as good as the best solution obtained through a traditional split-delivery vehicle routing problem with time windows.

But this study is not without its limitations. A future direction that requires further investigation is that of the effects of customer consolidation capability on shipment consolidation performance. Figure 16 shows that, even though savings were attained in general through consolidation, more savings were realized when $A_m=0$ for 33% of the customers than when all customers were consolidation capable ($A_m=1$ for all m). This experiment did not include paired analyses at varying levels of A_m, thus conclusions cannot readily be drawn. Though theoretically possible that the random instances generated did better when only 66% of the customers were capable of consolidation, rather than when 100% of them were, further investigation is needed. An extended study should determine whether it is possibly to establish a recommended number of customers that should, in general, support consolidation, or whether certain types of customers are better candidates for consolidation.

This study assessed the concerns of industry practitioners and offers recommendations to help firms determine whether shipment consolidation is likely to offer increased performance. Due to computational limitations, test instances remained small. Further studies can focus on the effects of the proposed model for larger problems through the adaptation of heuristics to reach feasible solutions within reasonable times.

APPENDIX: ADDITIONAL MATERIAL

Table 14 - Performance gap of the VRPC *vs.* the VRPSD by problem size and cell.

			VRPSD		VRPC		
Nodes	**Cell**	**Instances**	**Time (Min)**	**Average SD**	**Time (Min)**	**Average XD**	**Gap**
5	2	15	0.00	1.80	0.00	0.73	-3.8%
	10	15	0.00	1.67	0.00	0.80	-3.0%
	11	15	0.00	2.40	0.00	0.20	-2.1%
	14	15	0.00	1.13	0.00	0.33	-1.8%
	5	15	0.00	2.20	0.00	0.27	-1.5%
	9	15	0.00	2.80	0.00	1.20	-0.5%
	6	15	0.00	1.33	0.00	0.33	-0.3%
	8	15	0.00	0.80	0.00	0.00	-0.3%
	1	15	0.00	3.20	0.00	0.60	-0.2%
	4	15	0.00	1.87	0.00	0.07	-0.2%
	15	15	0.00	1.27	0.00	0.27	-0.1%
	16	15	0.00	0.87	0.00	0.00	0.1%
	12	15	0.00	1.33	0.00	0.27	0.1%
	13	15	0.00	2.07	0.00	0.33	0.4%
	7	15	0.00	1.93	0.00	0.20	0.5%
	3	15	0.00	2.00	0.00	0.20	1.9%
5 Total		**240**	**0.00**	**1.79**	**0.00**	**0.36**	**-0.7%**
6	10	15	0.01	3.00	0.01	0.80	-4.4%
	12	15	0.01	2.13	0.01	0.27	-2.5%
	9	15	0.02	3.73	0.01	1.13	-1.8%
	3	15	0.01	3.27	0.01	1.07	-1.8%
	13	15	0.01	3.20	0.01	0.60	-1.7%
	11	15	0.01	2.20	0.01	0.53	-1.2%
	4	15	0.01	3.13	0.01	0.80	-0.7%
	16	15	0.00	1.33	0.00	0.00	-0.4%
	1	15	0.02	3.60	0.01	0.73	-0.4%
	5	15	0.02	3.13	0.02	0.73	-0.1%
	8	15	0.00	1.73	0.00	0.00	0.0%
	7	15	0.00	2.20	0.00	0.07	0.0%
	2	15	0.18	3.07	0.01	1.33	0.0%
	15	15	0.00	1.80	0.00	0.13	0.0%
	14	15	0.01	2.73	0.01	0.93	0.1%
	6	15	0.02	2.73	0.01	1.20	2.7%
6 Total		**240**	**0.02**	**2.69**	**0.01**	**0.65**	**-0.8%**
7	8	15	0.01	2.40	0.01	0.40	-4.5%
	10	15	0.48	4.00	0.04	0.47	-4.2%
	12	15	0.05	3.00	0.02	0.53	-3.9%
	14	15	0.02	3.07	0.02	0.87	-2.9%
	2	15	0.13	3.67	0.05	1.53	-2.0%

Table 14 *continued*

Nodes	Cell	Instances	VRPSD Time (Min)	VRPSD Average SD	VRPC Time (Min)	VRPC Average XD	Gap
	4	15	0.04	3.47	0.03	0.67	-2.0%
	13	15	0.05	3.60	0.03	0.60	-1.4%
	6	15	0.04	3.47	0.04	1.47	-1.2%
	9	15	0.03	4.33	0.08	0.87	-1.1%
	15	15	0.01	2.60	0.01	0.20	-0.3%
	16	15	0.01	1.27	0.01	0.00	-0.3%
	5	15	0.07	4.27	0.07	1.07	-0.1%
	11	15	0.03	3.60	0.02	0.53	0.1%
	7	15	0.02	3.53	0.05	0.80	0.1%
	1	15	2.25	4.80	0.13	2.33	1.0%
	3	15	0.23	3.73	0.05	0.80	1.5%
7 Total		**240**	**0.22**	**3.43**	**0.04**	**0.82**	**-1.3%**
8	10	15	3.45	4.73	0.13	2.27	-6.3%
	14	15	0.08	3.40	0.04	0.87	-4.9%
	12	15	0.09	3.73	0.05	0.40	-2.6%
	3	15	0.63	4.80	0.23	2.00	-2.4%
	2	15	0.35	5.47	0.20	2.33	-1.7%
	6	15	0.16	4.27	0.10	0.80	-1.1%
	1	15	1.14	5.60	0.26	3.20	-1.1%
	16	15	0.02	2.47	0.02	0.13	-1.0%
	13	15	1.28	4.33	0.50	1.13	-0.9%
	9	15	10.75	5.93	3.45	2.07	-0.7%
	11	15	0.12	4.47	0.22	1.00	-0.3%
	8	15	0.03	4.13	0.03	0.73	-0.3%
	5	15	1.20	5.47	0.26	1.20	-0.3%
	7	15	0.17	4.33	0.16	0.40	-0.2%
	15	15	0.05	3.87	0.03	0.93	0.0%
	4	15	0.27	4.33	0.17	1.27	1.2%
8 Total		**240**	**1.24**	**4.46**	**0.36**	**1.30**	**-1.4%**
9	10	15	19.79	5.53	1.22	3.00	-9.6%
	12	15	4.83	4.87	0.98	1.47	-6.0%
	14	15	4.22	4.13	2.29	1.53	-5.6%
	6	15	0.40	5.33	0.41	2.20	-1.9%
	15	15	5.50	3.67	1.15	0.13	-1.6%
	9	15	15.24	5.73	6.93	1.67	-1.6%
	16	15	0.13	3.73	0.10	0.27	-1.5%
	11	15	18.61	5.73	12.30	1.20	-1.4%
	13	12	16.92	6.17	4.09	1.08	-1.3%
	2	15	2.69	6.60	1.82	2.80	-1.2%
	1	15	10.45	6.47	2.34	3.33	-0.8%
	5	15	5.49	5.40	3.00	2.00	-0.7%
	3	15	4.49	5.87	2.27	1.33	-0.1%

Table 14 *continued*

			VRPSD		VRPC		
Nodes	**Cell**	**Instances**	**Time (Min)**	**Average SD**	**Time (Min)**	**Average XD**	**Gap**
	4	15	1.04	5.20	0.42	1.60	0.0%
	7	15	2.96	4.80	2.37	1.13	1.2%
	8	15	0.17	4.47	0.07	0.80	2.6%
9 Total		**237**	**6.93**	**5.22**	**2.59**	**1.60**	**-1.9%**
10	10	15	16.89	6.80	3.91	1.87	-8.2%
	12	15	12.94	6.53	3.02	1.80	-6.2%
	14	15	11.79	5.33	1.06	1.67	-5.9%
	16	15	0.97	4.13	0.11	0.87	-3.3%
	9	15	28.52	7.53	24.03	2.27	-2.3%
	11	15	20.64	6.67	14.84	1.33	-1.8%
	2	15	7.52	6.87	10.41	4.00	-1.5%
	6	15	1.94	6.13	1.57	2.47	-1.5%
	4	15	7.05	5.53	5.06	3.13	-1.4%
	15	15	8.58	4.53	5.08	0.47	-1.4%
	13	15	12.37	6.13	9.36	2.47	-0.9%
	7	15	13.76	6.40	10.22	1.13	-0.3%
	1	15	9.49	7.47	2.70	3.67	-0.1%
	5	15	7.15	6.87	5.09	2.13	-0.1%
	8	15	0.86	5.67	0.26	0.80	0.0%
	3	15	6.08	7.20	2.17	3.73	0.7%
10 Total		**240**	**10.41**	**6.24**	**6.18**	**2.11**	**-2.1%**
Grand Total		**1437**	**3.13**	**3.97**	**1.53**	**1.14**	**-1.4%**

Notes: SD and XD is the number of nodes in which a split delivery and consolidation takes place, respectively. A more negative gap implies better performance

Table 15 - Results of the VRPCTW *vs*. VRPSDTW with 5 nodes

Cell	σ(arc length)	σ(demand P_1)	σ(demand P_2)	Fleet	Am Percent	tw Width	Time (Min)	Average SD	Average XD	Gap Obj value
1	50	200	200	0	0.667	50	0.04	2.20	0.07	-2.90%
2	50	200	200	0	0.667	100	0.13	2.07	0.00	0.00%
3	50	200	200	0	0.333	50	0.02	1.47	0.07	-0.24%
4	50	200	200	0	0.333	100	0.03	1.87	0.07	-0.59%
5	50	200	200	0	0	50	0.03	1.93	0.00	0.00%
6	50	200	200	0	0	100	0.06	2.13	0.00	0.00%
7	50	200	200	1	0.667	50	0.01	1.20	0.00	0.00%
8	50	200	200	1	0.667	100	0.02	0.93	0.07	-0.02%
9	50	200	200	1	0.333	50	0.06	0.80	0.27	-0.02%
10	50	200	200	1	0.333	100	0.02	0.73	0.00	0.00%
11	50	200	200	1	0	50	0.01	0.73	0.07	-0.09%
12	50	200	200	1	0	100	0.11	1.20	0.20	-0.10%
13	50	200	20	0	0.667	50	0.01	0.93	0.00	0.00%
14	50	200	20	0	0.667	100	0.01	1.27	0.00	0.00%
15	50	200	20	0	0.333	50	0.02	1.27	0.00	0.00%
16	50	200	20	0	0.333	100	0.01	1.53	0.00	0.00%
17	50	200	20	0	0	50	0.01	1.13	0.00	0.00%
18	50	200	20	0	0	100	0.01	1.00	0.00	0.00%
19	50	200	20	1	0.667	50	0.00	0.20	0.00	0.00%
20	50	200	20	1	0.667	100	0.01	0.47	0.00	0.00%
21	50	200	20	1	0.333	50	0.01	0.20	0.00	0.00%
22	50	200	20	1	0.333	100	0.01	0.33	0.00	0.00%
23	50	200	20	1	0	50	0.01	0.33	0.00	0.00%
24	50	200	20	1	0	100	0.00	0.33	0.07	-0.08%
25	50	20	200	0	0.667	50	0.01	1.00	0.00	0.00%
26	50	20	200	0	0.667	100	0.01	1.40	0.00	0.00%
27	50	20	200	0	0.333	50	0.02	1.40	0.00	0.00%
28	50	20	200	0	0.333	100	0.01	1.27	0.00	0.00%
29	50	20	200	0	0	50	0.01	0.80	0.00	0.00%
30	50	20	200	0	0	100	0.02	1.07	0.00	0.00%
31	50	20	200	1	0.667	50	0.01	0.40	0.00	0.00%
32	50	20	200	1	0.667	100	0.01	0.13	0.00	0.00%
33	50	20	200	1	0.333	50	0.01	0.40	0.07	-0.03%
34	50	20	200	1	0.333	100	0.01	0.20	0.00	0.00%
35	50	20	200	1	0	50	0.00	0.07	0.00	0.00%
36	50	20	200	1	0	100	0.01	0.20	0.00	0.00%

Table 15 *continued*

Cell	σ(arc length)	σ(demand P_1)	σ(demand P_2)	Fleet	Am Percent	tw Width	Time (Min)	Average SD	Average XD	Gap Obj value
37	50	20	20	0	0.667	50	0.00	0.13	0.00	0.00%
38	50	20	20	0	0.667	100	0.00	0.13	0.00	0.00%
39	50	20	20	0	0.333	50	0.00	0.00	0.00	0.00%
40	50	20	20	0	0.333	100	0.00	0.33	0.00	0.00%
41	50	20	20	0	0	50	0.00	0.13	0.00	0.00%
42	50	20	20	0	0	100	0.00	0.13	0.00	0.00%
43	50	20	20	1	0.667	50	0.00	0.00	0.00	0.00%
44	50	20	20	1	0.667	100	0.00	0.00	0.00	0.00%
45	50	20	20	1	0.333	50	0.00	0.00	0.00	0.00%
46	50	20	20	1	0.333	100	0.00	0.00	0.00	0.00%
47	50	20	20	1	0	50	0.00	0.00	0.00	0.00%
48	50	20	20	1	0	100	0.00	0.00	0.00	0.00%
49	1000	200	200	0	0.667	50	0.02	1.87	0.13	-0.53%
50	1000	200	200	0	0.667	100	0.02	2.27	0.00	0.00%
51	1000	200	200	0	0.333	50	0.02	1.80	0.00	0.00%
52	1000	200	200	0	0.333	100	0.01	1.93	0.07	-1.64%
53	1000	200	200	0	0	50	0.03	1.80	0.00	0.00%
54	1000	200	200	0	0	100	0.01	2.27	0.00	0.00%
55	1000	200	200	1	0.667	50	0.01	0.80	0.00	0.00%
56	1000	200	200	1	0.667	100	0.02	1.00	0.13	-0.44%
57	1000	200	200	1	0.333	50	0.01	0.93	0.33	-1.95%
58	1000	200	200	1	0.333	100	0.00	1.67	0.00	0.00%
59	1000	200	200	1	0	50	0.01	1.00	0.00	0.00%
60	1000	200	200	1	0	100	0.01	1.87	0.00	0.00%
61	1000	200	20	0	0.667	50	0.01	0.93	0.00	0.00%
62	1000	200	20	0	0.667	100	0.01	1.73	0.00	0.00%
63	1000	200	20	0	0.333	50	0.01	0.73	0.00	0.00%
64	1000	200	20	0	0.333	100	0.01	1.27	0.00	0.00%
65	1000	200	20	0	0	50	0.01	0.93	0.00	0.00%
66	1000	200	20	0	0	100	0.00	1.27	0.00	0.00%
67	1000	200	20	1	0.667	50	0.00	0.07	0.00	0.00%
68	1000	200	20	1	0.667	100	0.00	0.80	0.00	0.00%
69	1000	200	20	1	0.333	50	0.00	0.13	0.07	-0.39%
70	1000	200	20	1	0.333	100	0.00	1.27	0.00	0.00%
71	1000	200	20	1	0	50	0.00	0.13	0.13	-1.23%
72	1000	200	20	1	0	100	0.01	1.07	0.00	0.00%
73	1000	20	200	0	0.667	50	0.01	1.13	0.00	0.00%

Table 15 ***continued***

Cell	σ(arc length)	σ(demand P_1)	σ(demand P_2)	Fleet	Am Percent	tw Width	Time (Min)	Average SD	Average XD	Gap Obj value
74	1000	20	200	0	0.667	100	0.01	2.07	0.00	0.00%
75	1000	20	200	0	0.333	50	0.01	1.20	0.00	0.00%
76	1000	20	200	0	0.333	100	0.00	1.87	0.00	0.00%
77	1000	20	200	0	0	50	0.01	0.93	0.07	-0.06%
78	1000	20	200	0	0	100	0.01	1.33	0.00	0.00%
79	1000	20	200	1	0.667	50	0.55	0.27	0.00	0.00%
80	1000	20	200	1	0.667	100	0.00	1.27	0.00	0.00%
81	1000	20	200	1	0.333	50	0.00	0.33	0.00	0.00%
82	1000	20	200	1	0.333	100	0.00	1.47	0.00	0.00%
83	1000	20	200	1	0	50	0.00	0.13	0.00	0.00%
84	1000	20	200	1	0	100	0.00	1.33	0.20	-0.15%
85	1000	20	20	0	0.667	50	0.00	0.53	0.00	0.00%
86	1000	20	20	0	0.667	100	0.00	0.80	0.00	0.00%
87	1000	20	20	0	0.333	50	0.00	0.40	0.00	0.00%
88	1000	20	20	0	0.333	100	0.00	1.73	0.00	0.00%
89	1000	20	20	0	0	50	0.00	0.00	0.00	0.00%
90	1000	20	20	0	0	100	0.00	1.07	0.07	-0.40%
91	1000	20	20	1	0.667	50	0.00	0.00	0.00	0.00%
92	1000	20	20	1	0.667	100	0.00	0.20	0.00	0.00%
93	1000	20	20	1	0.333	50	0.00	0.00	0.00	0.00%
94	1000	20	20	1	0.333	100	0.00	1.00	0.00	0.00%
95	1000	20	20	1	0	50	0.00	0.00	0.00	0.00%
96	1000	20	20	1	0	100	0.00	0.40	0.00	0.00%

Notes: SD and XD is the number of nodes in which a split delivery and consolidation takes place, respectively.
A more negative gap implies better performance
Fleet is equal to one when the generated problem had a heterogeneous fleet, homogeneous otherwise

Table 16 - Results of the VRPCTW *vs*. VRPSDTW with 6 nodes

Cell	σ(arc length)	σ(demand P_1)	σ(demand P_2)	Fleet	Am Percent	tw Width	Time (Min)	Average SD	Average XD	Gap Obj value
1	50	200	200	0	0.667	50	0.96	2.67	0.07	-0.02%
2	50	200	200	0	0.667	100	1.51	2.67	0.00	0.00%
3	50	200	200	0	0.333	50	1.09	2.60	0.20	-4.79%
4	50	200	200	0	0.333	100	0.86	2.67	0.00	0.00%
5	50	200	200	0	0	50	2.22	2.40	0.07	-0.08%
6	50	200	200	0	0	100	1.66	2.47	0.20	-0.06%
7	50	200	200	1	0.667	50	2.47	1.87	0.00	0.00%
8	50	200	200	1	0.667	100	1.60	1.73	0.00	0.00%
9	50	200	200	1	0.333	50	1.11	1.20	0.07	-1.48%
10	50	200	200	1	0.333	100	1.32	1.73	0.07	-0.01%
11	50	200	200	1	0	50	2.08	2.07	0.33	-0.12%
12	50	200	200	1	0	100	1.24	1.87	0.20	-0.96%
13	50	200	20	0	0.667	50	0.49	1.33	0.00	0.00%
14	50	200	20	0	0.667	100	1.02	1.27	0.00	0.00%
15	50	200	20	0	0.333	50	0.95	1.93	0.07	-0.08%
16	50	200	20	0	0.333	100	0.67	1.47	0.00	0.00%
17	50	200	20	0	0	50	1.11	1.73	0.13	-2.36%
18	50	200	20	0	0	100	0.37	1.33	0.07	-3.94%
19	50	200	20	1	0.667	50	0.27	0.87	0.00	0.00%
20	50	200	20	1	0.667	100	0.10	0.47	0.00	0.00%
21	50	200	20	1	0.333	50	0.45	0.27	0.00	0.00%
22	50	200	20	1	0.333	100	0.42	0.60	0.00	0.00%
23	50	200	20	1	0	50	0.21	0.67	0.00	0.00%
24	50	200	20	1	0	100	0.16	0.67	0.07	-0.10%
25	50	20	200	0	0.667	50	0.31	1.53	0.00	0.00%
26	50	20	200	0	0.667	100	0.26	1.33	0.00	0.00%
27	50	20	200	0	0.333	50	0.43	1.47	0.07	-0.67%
28	50	20	200	0	0.333	100	1.10	1.33	0.00	0.00%
29	50	20	200	0	0	50	1.43	1.33	0.07	-0.25%
30	50	20	200	0	0	100	0.17	1.33	0.00	0.00%
31	50	20	200	1	0.667	50	0.20	0.67	0.00	0.00%
32	50	20	200	1	0.667	100	0.21	0.60	0.07	-0.03%
33	50	20	200	1	0.333	50	1.06	0.93	0.07	-0.01%
34	50	20	200	1	0.333	100	0.26	1.07	0.00	0.00%
35	50	20	200	1	0	50	0.30	0.87	0.07	-0.03%
36	50	20	200	1	0	100	0.97	0.93	0.07	-0.15%

Table 16 *continued*

Cell	σ(arc length)	σ(demand P_1)	σ(demand P_2)	Fleet	Am Percent	tw Width	Time (Min)	Average SD	Average XD	Gap Obj value
37	50	20	20	0	0.667	50	0.08	0.47	0.13	-1.30%
38	50	20	20	0	0.667	100	0.02	0.53	0.00	0.00%
39	50	20	20	0	0.333	50	0.01	0.27	0.13	-0.04%
40	50	20	20	0	0.333	100	0.05	0.27	0.00	0.00%
41	50	20	20	0	0	50	0.02	0.40	0.20	-4.44%
42	50	20	20	0	0	100	0.02	0.53	0.20	-10.48%
43	50	20	20	1	0.667	50	0.01	0.00	0.00	0.00%
44	50	20	20	1	0.667	100	0.07	0.00	0.00	0.00%
45	50	20	20	1	0.333	50	0.01	0.07	0.00	0.00%
46	50	20	20	1	0.333	100	0.01	0.07	0.00	0.00%
47	50	20	20	1	0	50	0.01	0.00	0.00	0.00%
48	50	20	20	1	0	100	0.02	0.00	0.00	0.00%
49	1000	200	200	0	0.667	50	0.86	2.47	0.00	0.00%
50	1000	200	200	0	0.667	100	0.65	2.93	0.00	0.00%
51	1000	200	200	0	0.333	50	1.39	2.20	0.00	0.00%
52	1000	200	200	0	0.333	100	0.16	3.13	0.00	0.00%
53	1000	200	200	0	0	50	1.23	2.13	0.00	0.00%
54	1000	200	200	0	0	100	1.27	3.00	0.00	0.00%
55	1000	200	200	1	0.667	50	0.23	1.27	0.13	-1.23%
56	1000	200	200	1	0.667	100	0.23	2.20	0.00	0.00%
57	1000	200	200	1	0.333	50	0.25	1.53	0.47	-1.10%
58	1000	200	200	1	0.333	100	0.45	2.27	0.33	-0.87%
59	1000	200	200	1	0	50	0.75	1.80	0.47	-2.62%
60	1000	200	200	1	0	100	0.36	2.47	0.07	-0.23%
61	1000	200	20	0	0.667	50	0.58	1.60	0.00	0.00%
62	1000	200	20	0	0.667	100	0.18	2.47	0.00	0.00%
63	1000	200	20	0	0.333	50	0.45	1.87	0.00	0.00%
64	1000	200	20	0	0.333	100	0.44	2.07	0.00	0.00%
65	1000	200	20	0	0	50	0.09	1.33	0.00	0.00%
66	1000	200	20	0	0	100	0.21	2.53	0.00	0.00%
67	1000	200	20	1	0.667	50	0.04	0.67	0.00	0.00%
68	1000	200	20	1	0.667	100	0.08	2.07	0.00	0.00%
69	1000	200	20	1	0.333	50	0.07	0.53	0.13	-0.05%
70	1000	200	20	1	0.333	100	0.08	2.07	0.00	0.00%
71	1000	200	20	1	0	50	0.05	0.60	0.00	0.00%
72	1000	200	20	1	0	100	0.20	1.40	0.20	-1.65%
73	1000	20	200	0	0.667	50	0.28	1.40	0.00	0.00%

Table 16 ***continued***

Cell	σ(arc length)	σ(demand P_1)	σ(demand P_2)	Fleet	Am Percent	tw Width	Time (Min)	Average SD	Average XD	Gap Obj value
74	1000	20	200	0	0.667	100	0.69	2.47	0.00	0.00%
75	1000	20	200	0	0.333	50	0.39	1.40	0.07	-0.35%
76	1000	20	200	0	0.333	100	0.31	2.47	0.00	0.00%
77	1000	20	200	0	0	50	0.21	1.60	0.27	-2.23%
78	1000	20	200	0	0	100	0.03	2.07	0.13	-1.04%
79	1000	20	200	1	0.667	50	0.05	0.73	0.07	-0.96%
80	1000	20	200	1	0.667	100	0.14	2.00	0.00	0.00%
81	1000	20	200	1	0.333	50	0.04	0.47	0.07	-1.10%
82	1000	20	200	1	0.333	100	0.03	2.07	0.00	0.00%
83	1000	20	200	1	0	50	0.05	0.67	0.00	0.00%
84	1000	20	200	1	0	100	0.02	2.20	0.00	0.00%
85	1000	20	20	0	0.667	50	0.01	0.47	0.07	-1.63%
86	1000	20	20	0	0.667	100	0.01	2.07	0.13	-0.15%
87	1000	20	20	0	0.333	50	0.01	0.47	0.00	0.00%
88	1000	20	20	0	0.333	100	0.01	2.47	0.00	0.00%
89	1000	20	20	0	0	50	0.01	0.13	0.13	-2.74%
90	1000	20	20	0	0	100	0.02	1.93	0.27	-2.48%
91	1000	20	20	1	0.667	50	0.00	0.00	0.00	0.00%
92	1000	20	20	1	0.667	100	0.01	1.60	0.00	0.00%
93	1000	20	20	1	0.333	50	0.00	0.00	0.00	0.00%
94	1000	20	20	1	0.333	100	0.01	1.60	0.00	0.00%
95	1000	20	20	1	0	50	0.01	0.00	0.00	0.00%
96	1000	20	20	1	0	100	0.01	1.33	0.00	0.00%

Notes: SD and XD is the number of nodes in which a split delivery and consolidation takes place, respectively.
A more negative gap implies better performance
Fleet is equal to one when the generated problem had a heterogeneous fleet, homogeneous otherwise

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